

Banco de México

Working Papers

N° 2022-20

Foreign Currency Working Capital Constraints for  
Imported Inputs and Compositional Effects in  
Intermediate Goods

Daniel Sámano

Banco de México

December 2022

La serie de Documentos de Investigación del Banco de México divulga resultados preliminares de trabajos de investigación económica realizados en el Banco de México con la finalidad de propiciar el intercambio y debate de ideas. El contenido de los Documentos de Investigación, así como las conclusiones que de ellos se derivan, son responsabilidad exclusiva de los autores y no reflejan necesariamente las del Banco de México.

The Working Papers series of Banco de México disseminates preliminary results of economic research conducted at Banco de México in order to promote the exchange and debate of ideas. The views and conclusions presented in the Working Papers are exclusively the responsibility of the authors and do not necessarily reflect those of Banco de México.

# Foreign Currency Working Capital Constraints for Imported Inputs and Compositional Effects in Intermediate Goods\*

Daniel Sámano<sup>†</sup>  
Banco de México

**Abstract:** I develop an asymmetric two-country incomplete markets model in which economies trade final consumption goods and inputs. The purchases of imported inputs from the firms of one of the economies (the emerging) to the firms of the other economy (the advanced) are subject to a foreign currency working capital constraint. Domestic firms are assumed to finance their working capital by borrowing from the domestic household in local currency. Through numerical simulations, I show that in this environment domestic productivity shocks have compositional effects through the cost of the working capital. In particular, after a domestic positive productivity shock terms of trade rise and the working capital cost exhibits a sudden increase followed by a prolonged temporary decrease. This leads to inputs recomposition in the domestic economy in response to working capital cost adjustments.

**Keywords:** Working capital, Foreign currency, Imported inputs

**JEL Classification:** C68, F15, F41

**Resumen:** Se desarrolla un modelo de mercados incompletos de dos países asimétricos con mercados incompletos en el que las economías intercambian bienes de consumo final e insumos. Las compras de insumos importados de las firmas de una de las economías (la emergente) a las firmas de la otra economía (la avanzada) están sujetas a una restricción de capital de trabajo en moneda extranjera. Se supone que las firmas domésticas financian su capital de trabajo mediante préstamos del hogar doméstico en moneda local. A través de simulaciones numéricas, se muestra que en este entorno los choques de productividad domésticos tienen efectos de recomposición a través del costo del capital de trabajo. En particular, tras un choque positivo de productividad doméstica, los términos de intercambio se aprecian y el costo del capital de trabajo exhibe un aumento repentino seguido de una disminución temporal prolongada. Esto lleva a una recomposición de insumos en la economía emergente en respuesta a los ajustes en el costo del capital de trabajo.

**Palabras Clave:** Capital de trabajo, Moneda extranjera, Insumos importados

---

\*I thank two anonymous referees for their very enriching comments and suggestions. All errors and omissions remain my own.

<sup>†</sup> Dirección General de Investigación Económica. Email: dsamano@banxico.org.mx.

# 1 Introduction

After the Great Financial Crisis (GFC) the linkages between the real economy and the financial one became very apparent. Trade activity was not the exception. By matching exports data from Japan with that of the institutions that provided trade finance from 1990 to 2010, Amiti and Weinstein (2011) offer compelling evidence of how this could be explained in part due to the deterioration in trade finance. Similar reasoning is followed by Chor and Manova (2012) based on data for several economies and the U.S. imports from November 2006 to October 2009. By merging a disaggregated data set of more than 50,000 exports flows from July 2007 to June 2009 that matches Peruvian exports by firm and product to multiple destinations with local bank loans, Paravisini, Rappoport, Schnabl, and Wolfenzon (2015) find that approximately 8% of the drop in the volume of exports can be attributed to the credit shock.

Klapper, Laeven, and Rajan (2012) provide evidence of ample arrangements of trade credit between buyers and sellers and the role it plays as a verification mechanism of the quality of goods exchanged through the length of the contract and as a way to discriminate prices (see also Petersen and Rajan (1995)). The work of Antràs and Foley (2015) can also be categorized in this line of work. They document the sales of a U.S. based firm that exports frozen and refrigerated food products, primarily poultry to more than 140 countries over the 1996-2009 period. In particular, they report many “cash in advance” terms. These financing terms imply that before the export arrives to its destination, 100% must have been covered either by a wire transfer and/or a deposit. They also provide a partial equilibrium model to explain the differences in how trade is financed depending on the contractual enforcement conditions of the exporting and importing countries. That model incorporates, among other things, some dynamics into the one of Schmidt-Eisenlohr (2013). This body of literature is for the most part empirical as it attempts to shed light on the enormous number of contracts available in international trade practices and the role they play. The one that is emphasized by Klapper, Laeven, and Rajan (2012) is that of a verification mechanism while Antràs and Foley (2015) and Schmidt-Eisenlohr (2013) place emphasis on contractual enforcement.

In terms of macroeconomic models, working capital frictions are used to align the canonical small open economy (SOE, hereafter) model to the stylized facts of these economies as shown by Neumeyer and Perri (2005), Uribe and Yue (2006) and Mendoza and Yue (2012). In particular, the working capital friction on a share of the wage bill in the former two cases and on imported inputs in the latter produce a more volatile business cycles with respect to the one of advanced economies.

This paper attempts to contribute to the existing literature of general equilibrium models of international economics that incorporate financial frictions, albeit stylized, particularly some that are likely to be present in emerging market economies vis-à-vis advanced ones. I depart from the SOE framework and adapt the production block of Mendoza and Yue (2012) into a two-country model economy to obtain endogenous prices, among them, terms of trade, the real and nominal exchange rate and nominal and real interest rates in an asymmetric setting (in terms of working population size) in order to obtain an *endogenous* working capital cost faced by firms in the emerging market economy.<sup>1</sup> By doing this, I attempt to answer the following question: what is the role of working capital costs in foreign currency in an emerging market economy within a setting of prototype economies that share production processes? The production sharing in this environment means that each economy has a composite intermediate good comprised of domestic and imported inputs to produce together with labor consumption goods that can be either consumed domestically or abroad.

Apart from endogenizing the working capital cost, this modeling choice was made since I am interested in potential feedback effects from shocks occurring in one of the economies (for instance, the domestic) to the other economy (the advanced one) and how they propagate in general equilibrium. In particular, the working capital constraint that I assume takes the form of a payment in advance constraint for imported inputs faced by domestic economy firms as in Mendoza and Yue (2012). Although domestic firms sell to the local and the foreign market, we assume that every period they borrow in domestic currency on an intra period basis from the local household the locally denominated funds that later on they exchange to meet working capital requirements in foreign currency. This makes the working capital cost for imported inputs a function of the domestic interest rate on intratemporal loans times the share of the imported inputs bill that must be paid in advance.<sup>2</sup> Importantly, we assume that firms in the emerging economy are subject to this constraint but not those in the advanced economy. In other words, firms in the home ( $H$ ) economy (the emerging one) are subject to a working capital constraint in the currency of the exporter for a share of the bill of imported intermediate goods from the foreign ( $F$ ) economy (the advanced one). For the same type of purchase,  $F$ -economy firms obtain credit from exporters, which means that they can pay for imported inputs after production takes place. Although exogenously imposed in this setting,

---

<sup>1</sup>In this regard, my work also departs from the economies of Neumeyer and Perri (2005) and Uribe and Yue (2006).

<sup>2</sup>When this share is zero, which would imply that domestic firms do not face a working capital constraint for their imported inputs, the working capital cost is simply null. On the other hand, when the share is one, which would imply that the full imported inputs bill must be paid in advance, then the working capital cost is equivalent to the interest rate on intra period loans.

the payment in advance constraint is more likely to be present in environments where contract enforcement is imperfect, as Antràs and Foley (2015) find.<sup>3</sup> Therefore, I try to represent a setting in which emerging market economy firms face commitment problems to honor their payments in international purchases of inputs. As mentioned before, labor and a bundle of imported inputs and domestic ones are used by local firms to produce final goods that can be consumed or exported.

Numerical simulations indicate that  $H$ -economy productivity shocks induce endogenous variations in terms of trade. In particular, after a domestic positive productivity shock terms of trade exhibit a temporary appreciation from the perspective of the  $H$ -economy. The shock also generates a sudden increase in the interest rate on intra period loans demanded by  $H$ -economy firms followed by a prolonged temporary decrease. This translates into a proportional adjustment in the working capital cost and therefore, to inputs recomposition in the domestic economy, which at the margin demands more imported inputs vis-à-vis domestic ones when working capital requirements are higher. Due to general equilibrium effects allocations of the  $F$ -economy are also affected.

Hence, the model introduces a mechanism through which a relatively simple financial friction allows for the interaction of terms of trade and the working capital cost. The result of this interplay is the equilibrium inputs' mix composition in the economy that faces a payment in advance constraint for imported inputs. Due to general equilibrium effects corresponding allocations of the foreign economy are also affected.

I recognize that two country models such as those of the New Open Economy Macroeconomics<sup>4</sup> and International Real Business Cycles (see Backus, Kehoe, and Kydland (1992)) literature have traditionally been used to study economies with similar characteristics, usually advanced ones. Nonetheless, I have departed from the SOE framework to study the Mexican economy vis-à-vis the U.S. given their production integration, which admittedly is yet simple in the current framework. In this sense, the results should be seen merely as illustrative. However, this approach is useful, among other scenarios, when one is interested in endogenizing crucial prices. Leading examples on this line are the works of Chen and Crucini (2016) and Rothert (2020). The former work extends the framework developed in Baxter and Crucini

---

<sup>3</sup>They find that this type of contract exhibits some degree of stickiness for their sample of firms on an aggregate basis during the GFC. Nevertheless, adjustments in financing terms across firms played an important role for financing composition in such a period since new importers relied more on “cash in advance” terms while existing customers depended more on credit. For their entire sample of analysis, they also find that as the relationship between the exporting and importing firm evolves over time, parties involved tend to rely more on credit.

<sup>4</sup>See Lane (2001) for an excellent survey and Corsetti, Dedola, and Leduc (2008) for an outstanding application of this framework.

(1995) to consider the interaction of a SOE with a “rest of the world” economy while the latter introduces in such a framework consumption of a constant elasticity of substitution aggregator of consumption goods produced in the two economies, which allows endogenizing the real exchange rate.

## 2 The Model

The model economy consists of two asymmetric economies, the home ( $H$ ) and the foreign ( $F$ ) one, which trade two types of goods: i) country specific final consumption goods, and ii) country specific intermediate goods or inputs. Each country has a unit mass of identical firms that use labor and a composite intermediate good comprised of domestic and imported inputs to produce economy specific goods that can be either consumed or used as inputs domestically or abroad. The production structure is of a “roundabout” type, following Basu (1995), which implies that it is modeled as an input-output process. This means that there are no “first” intermediate and latter final goods.<sup>5</sup> In other words,  $H$  goods are used to produce  $F$  goods, but  $F$  goods are also used to produce  $H$  goods, while at the same time,  $H$  and  $F$  goods could be used up as final goods in either economy. This production structure is key for the results, however, it greatly simplifies the analysis since expressions for the prices of bundles of consumption goods and inputs are very similar and therefore, facilitate analytical tractability.

Domestic and foreign consumption goods and inputs are combined by a constant elasticity of substitution aggregator (CES) featuring home bias. Apart from being of different size (in terms of their working population), the economies differ in a few aspects although for the moment we will focus on the fact that purchases of imported inputs by firms of the  $H$ -economy are subject to a working capital requirement in the currency of the  $F$ -economy. In other words, domestic firms must pay a share of their imported intermediate goods bill before production takes place. The need to denominate the working capital requirements in foreign currency rather than in the domestic one captures the idea that prices are set at the producer currency and that international transactions are more difficult to enforce than domestic ones. Thus, such transactions need to be settled in advance (at least, a share of each purchase). We assume that domestic firms borrow on an intra period basis the funds to meet working capital requirements in foreign currency from the local household (through a financial intermediary).

---

<sup>5</sup>This production structure has been used by Itskhoki and Mukhin (2017) and by Itskhoki and Mukhin (2021) in the context of a two-country economy model and by Bouakez, Cardia, and Ruge-Murcia (2009) in a multi-sector monetary model for the U.S. economy.

The loan, however, is denominated in domestic currency. Therefore, an instant later firms exchange the borrowed funds for foreign currency with the domestic household. We assume this is the only source of financing they have available. This makes the working capital cost to depend on the domestic nominal interest rate on intra period loans. On the other hand, foreign firms are not subject to any working capital constraint. We can imagine they obtain their imported inputs using credit from domestic exporters and pay them back after production takes place.

In the model, financial markets are assumed to be incomplete across countries. We assume that in both economies, monetary authorities follow conventional Taylor rules as in Taylor (1993). This will allow us to focus on the role of the cost of the foreign currency working capital constraint in the  $H$ -economy shaping the equilibrium dynamics incorporating endogenous monetary responses.

## 2.1 Household's Problem in the Home Economy

Given  $B_{-1}, B_{-1}^*, Z_0^*, Z_0$  and  $\{e_t, W_t, i_t, i_t^*, i_t^d, P_t, \Pi_t, \{P_{H,t}(i)\}_{i \in [0,1]}, \{P_{F,t}^*(j)\}_{j \in [0,1]}\}$ , the representative household in the  $H$ -economy solves the following problem:

$$\max_{\{c_{H,t}, c_{F,t}, l_t, D_t, B_t^*, B_t, Z_{t+1}^*, Z_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\varphi(\bar{l} - l_t)^{1+\chi}}{1+\chi} \right) \quad (1)$$

s.t.

$$\begin{aligned} P_t c_t + \frac{D_t}{1+i_t^d} + e_t Z_{t+1}^* + Z_{t+1} + \frac{e_t B_t^*}{1+i_t^*} + \frac{B_t}{1+i_t} \\ = W_t l_t + D_t + e_t Z_t^* + Z_t + e_t B_{t-1}^* + B_{t-1} + \Pi_t \forall t \end{aligned} \quad (2)$$

where the latter equation is the  $H$ -economy representative household budget constraint and  $c_t$  is aggregate consumption,  $l_t$  is labor,  $\bar{l}$  is the total endowment of time each period which can be used for working or enjoying leisure,  $B_t$  ( $B_t^*$ ) are nominal holdings of bonds denominated in domestic (foreign) currency,  $D_t$  are domestic currency denominated intra period deposits,<sup>6</sup>  $Z_t^*$  are nominal foreign currency balances,  $Z_t$  are nominal domestic money balances,  $e_t$  is the nominal exchange rate (units of domestic currency per unit of foreign currency),  $W_t$  is the nominal wage,  $i_t$  ( $i_t^*$ ) is the net domestic (foreign) nominal interest rate on corresponding bonds,  $i_t^d$  is the net nominal interest rate on deposits and  $P_t$  is the domestic price level.

<sup>6</sup>We introduce intra period deposits to fund intra period loans to fund working capital in local currency as in Chowdhury, Hoffmann, and Schabert (2006) and Ida (2011), among others.

Allocations refer to those of the  $H$ -economy. Moreover,  $\Pi_t \equiv \int_0^1 \Pi_t(i) di$  (to be defined precisely below) are nominal profits from firms in the domestic economy. The restrictions in the parameters are that  $\beta < 1$ ,  $\sigma > 1$ ,  $\chi \geq 0$  and  $\varphi > 0$ . In addition, the household faces a cash-in-advance constraint as in Ida (2011) and similar to that of Christiano and Eichenbaum (1995) of the following form

$$P_t c_t \leq e_t Z_t^* + Z_t - \frac{D_t}{1 + i_t^d} \quad (3)$$

Notice that constraint (3) takes into account the fact that for aggregate consumption, the representative household exchanges foreign currency into domestic currency at the prevailing nominal exchange rate while it also uses domestic money balances and subtracts deposits made at the beginning of the period.

The composite consumption good is

$$c_t \equiv \left( \int_0^1 \left[ (1 - \omega)^{\frac{1}{\eta}} c_{H,t}(i)^{\frac{\eta-1}{\eta}} + \omega^{\frac{1}{\eta}} c_{F,t}(i)^{\frac{\eta-1}{\eta}} \right] di \right)^{\frac{\eta}{\eta-1}}, \eta > 1 \quad (4)$$

where  $c_{H,t}(i)$  is the consumption of the domestic good  $i \in [0, 1]$ ,  $c_{F,t}(j)$  is the consumption of the foreign good  $j \in [0, 1]$ , the parameter  $\omega < \frac{1}{2}$  captures the degree of home bias in consumption and the parameter  $\eta$  is the intratemporal elasticity of substitution (or elasticity of substitution between  $H$ - and  $F$ -produced goods). Consequently,

$$P_t \equiv \left( \int_0^1 \left[ (1 - \omega) P_{H,t}(i)^{1-\eta} + \omega (e_t P_{F,t}^*(i))^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}} \quad (5)$$

is the ideal domestic price index where  $P_{H,t}(i)$  is the price of domestic good  $i \in [0, 1]$  and  $P_{F,t}^*(j)$  is the price of foreign good  $j \in [0, 1]$  expressed in foreign currency which implies that  $\frac{P_{H,t}(i)}{P_t}$  is the relative price of the domestic good  $i$  and  $\frac{e_t P_{F,t}^*(j)}{P_t}$  is the relative price of the foreign good  $j$ . Therefore, in (5) we are assuming Producer Currency Pricing (PCP), so we are expressing the latter price in  $F$ -economy currency and then we are multiplying it by the nominal exchange rate,  $e_t P_{F,t}^*(j)$ .

The sequence of events for the representative household on a given period  $t$  occurs similar to Ida (2011) and Ida (2020).<sup>7</sup> The representative household enters period  $t$  with foreign cash holdings of  $Z_t^*$  and domestic money balances of  $Z_t$ . It exchanges with  $H$ -economy firms its foreign money balances for domestic ones at the prevailing nominal exchange rate  $e_t$ . The

<sup>7</sup>An important difference is that those articles assume that current wages can be used to finance current consumption.



household uses this cash to make deposits  $\frac{D_t}{1+i_t^d}$  at the financial intermediary. Remaining cash balances of  $e_t Z_t^* + Z_t - \frac{D_t}{1+i_t^d}$  are available to purchase consumption goods subject to a cash-in-advance constraint as in (3). After that, the household works and gets paid. Subsequently, it makes its decision on domestic and foreign currency denominated bonds holdings,  $\frac{B_t^*}{1+i_t^*}$  and  $\frac{B_t}{1+i_t}$ , while at the same time it receives  $B_{t-1}^*$  and  $B_{t-1}$  from one period before. At the end of the period, the household receives profits from firms and the principle plus interest on its deposits at the financial intermediary. As it will be seen below, the domestic representative household exchanges the principal of the deposit plus interest it receives at the end of the period for foreign currency with domestic firms. This is how the representative household enters the next period with a positive amount of foreign money balances.<sup>8</sup>

From the first order conditions of the household's problem we obtain (see Appendix A.1)

$$\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{(1+i_{t-1}^*) \left( \frac{e_{t+1}}{e_t} \right)}{\pi_{t+1}} \right] = 1 \quad (6)$$

$$\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{(1+i_{t-1})}{\pi_{t+1}} \right] = 1 \quad (7)$$

and

$$\frac{\varphi(\bar{l} - l_t)^\chi}{c_t^{-\sigma}} = \frac{w_t}{1+i_t^d} \quad (8)$$

where (6) is the Euler equation for bonds denominated in foreign currency, (7) is the Euler equation for bonds denominated in domestic currency and (8) is the labor supply. Notice that the latter condition is distorted due to the cash-in-advance constraint that the household faces.  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  is domestic gross inflation at time  $t$ ,  $w_t$  is the real wage and  $E_t[\cdot]$  is the expectation operator conditional on the information at time  $t$ . Moreover, condition

$$P_t c_t = e_t Z_t^* + Z_t - \frac{D_t}{(1+i_t^d)} \quad (9)$$

must hold every period. It can also be shown that  $i_t^d = i_{t-1} = i_{t-1}^*$ .

In addition to the consumption/savings and labor decisions of problem (1), taking as given relative prices  $\left\{ \frac{P_{H,t}(i)}{P_t} \right\}_{i \in [0,1]}$  and  $\left\{ \frac{e_t P_{F,t}(j)}{P_t} \right\}_{j \in [0,1]}$ , the household must decide each period

---

<sup>8</sup>From the point of view of domestic firms, as it will be seen below, they are required domestic currency at the end of the period to pay back an intra period loan denominated in domestic currency for working capital purposes, but they have foreign currency which is obtained from exporting firms when domestic firms liquidate the total amount of imported inputs bill at the end of the period. In other words, the payment in advance is returned plus interest in foreign currency when the total bill is paid at the end of the period.

how to split its consumption between domestic and foreign consumption goods. Optimal allocations of  $c_{H,t}(i)$  and  $c_{F,t}(j)$  are given, respectively, by

$$c_{H,t}(i) = (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\eta} c_t \quad \forall i \in [0, 1] \quad (10)$$

and

$$c_{F,t}(j) = \omega \left( \frac{e_t P_{F,t}^*(j)}{P_t} \right)^{-\eta} c_t \quad \forall j \in [0, 1] \quad (11)$$

where  $c_t$  satisfies

$$P_t c_t = \int_0^1 [P_{H,t}(i) c_{H,t}(i) + e_t P_{F,t}^*(i) c_{F,t}(i)] di \quad (12)$$

## 2.2 Household's Problem in the Foreign Economy

Taking as given  $\tilde{B}_{-1}^*$ ,  $\tilde{Z}_0^*$  and  $\left\{ e_t, W_t^*, i_t^*, i_t^{*L}, P_t^*, \Pi_t^*, \{P_{H,t}(i)\}_{i \in [0,1]}, \{P_{F,t}^*(j)\}_{j \in [0,1]} \right\}$  the representative foreign household solves the following problem

$$\max_{\{c_{H,t}^*, c_{F,t}^*, l_t^*, \tilde{B}_t^*, L_t^*, \tilde{Z}_{t+1}^*\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{*1-\sigma}}{1-\sigma} + \frac{\varphi^* (\bar{l} - l_t^*)^{1+\chi^*}}{1+\chi^*} \right) \quad (13)$$

s.t.

$$P_t^* c_t^* + \frac{\tilde{B}_t^*}{1+i_t^*} + \tilde{Z}_{t+1}^* - \frac{L_t^*}{(1+i_t^{*L})} = W_t^* l_t^* + \tilde{B}_{t-1}^* + \tilde{Z}_t^* - L_t^* + \Pi_t^* \forall t \quad (14)$$

and the cash-in-advance constraint of the form

$$P_t^* c_t^* \leq \tilde{Z}_t^* + \frac{L_t^*}{(1+i_t^{*L})} \quad (15)$$

where the composite consumption good is given by

$$c_t^* \equiv \left( \int_0^1 \left[ \omega^{*\frac{1}{\eta}} c_{H,t}^*(i)^{\frac{\eta-1}{\eta}} + (1-\omega^*)^{\frac{1}{\eta}} c_{F,t}^*(i)^{\frac{\eta-1}{\eta}} \right] di \right)^{\frac{\eta}{\eta-1}}, \eta > 1 \quad (16)$$

and where  $c_t^*$  is the foreign aggregate consumption,  $c_{H,t}^*(i)$  is the foreign consumption of the  $H$ -economy good  $i \in [0, 1]$ ,  $c_{F,t}^*(j)$  is the foreign consumption of the  $F$ -good  $j \in [0, 1]$ ,  $l_t^*$  is foreign labor,  $\bar{l}$  is time endowment,  $\tilde{B}_t^*$  are nominal bond holdings denominated in foreign

currency,  $\tilde{Z}_t^*$  are nominal holdings of foreign ( $F$ -economy) currency,  $L_t^*$  are foreign currency denominated intra period loans made by the foreign household to a local financial intermediary at a net nominal interest rate of  $i_t^{*L}$ ,  $W_t^*$  is the nominal wage,  $P_t^*$  is the foreign price level and  $\Pi_t^* \equiv \int_0^1 \Pi_t^*(j) dj$  are nominal profits of  $F$ -economy firms. The restrictions in the parameters are that  $\varphi^* > 0$ ,  $\chi^* \geq 0$  and  $\omega^* < \frac{1}{2}$ .

From the first order conditions of the foreign household's problem we obtain

$$\beta E_t \left[ \left( \frac{c_{t+1}^*}{c_t^*} \right)^{-\sigma} \frac{(1 + i_{t-1}^{*L})}{\pi_{t+1}^*} \right] = 1 \quad (17)$$

$$\frac{\varphi^* (\bar{l} - l_t^*)^{\chi^*}}{c_t^{*\sigma}} = \frac{w_t^*}{1 + i_t^{*L}} \quad (18)$$

$$P_t^* c_t^* = \tilde{Z}_t^* + \frac{L_t^*}{(1 + i_t^{*L})} \quad (19)$$

and

$$i_t^{*L} = i_{t-1}^{*L} \quad (20)$$

where  $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$  is the foreign gross inflation at time  $t$  and  $w_t^*$  is the real wage in the  $F$ -economy. Additionally, taking as given relative prices  $\left\{ \frac{P_{H,t}(i)}{e_t P_t^*} \right\}_{i \in [0,1]}$  and  $\left\{ \frac{P_{F,t}(j)}{P_t^*} \right\}_{j \in [0,1]}$ , the household must choose how to split its consumption between  $H$  and  $F$  consumption goods. Optimal allocations of  $c_{H,t}^*(i)$  and  $c_{F,t}^*(j)$  satisfy, respectively,

$$c_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}(i)}{e_t P_t^*} \right)^{-\eta} c_t^* \quad \forall i \in [0, 1] \quad (21)$$

$$c_{F,t}^*(j) = (1 - \omega^*) \left( \frac{P_{F,t}(j)}{P_t^*} \right)^{-\eta} c_t^* \quad \forall j \in [0, 1] \quad (22)$$

where the ideal price index in the  $F$ -economy is given by

$$P_t^* \equiv \int_0^1 \left( \left[ \omega^* \left( \frac{P_{H,t}(i)}{e_t} \right)^{1-\eta} + (1 - \omega^*) P_{F,t}^*(i)^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}} \quad (23)$$

where once again we are assuming PCP and  $P_t^* c_t^* = \int_0^1 \left[ \frac{P_{H,t}(i)}{e_t} c_{H,t}^*(i) + P_{F,t}^*(i) c_{F,t}^*(i) \right] di$  holds.

From the definition of ideal price indexes,  $Q_t$  satisfies in equilibrium

$$Q_t^{1-\eta} = \frac{\omega^* + (1 - \omega^*)T_t^{1-\eta}}{(1 - \omega) + \omega T_t^{1-\eta}} \quad (24)$$

### 2.3 *H*-Economy's Financial Intermediary

Following Chowdhury, Hoffmann, and Schabert (2006) and Ida (2011) we model a competitive representative financial intermediary that provides intra period deposits services to the *H*-economy household. It takes deposits from households and lends to domestic firms. The financial intermediary receives from the household  $\frac{D_t}{1+i_t^d}$  units of domestic currency at the beginning of every period and gives back  $D_t$  at the end of that period. For simplicity, we abstract from agency problems and other intermediation issues that in practice drive a wedge between the rate on deposits,  $i_t^d$ , and the rate on loans,  $i_t^L$ . The latter is the rate that domestic firms pay for loans in domestic currency to meet working capital requirements. Formally, the problem of the financial intermediary is

$$\max_{\{L_t, D_t\}} \frac{L_t}{1+i_t^L} - \frac{D_t}{1+i_t^d} \quad (25)$$

s.t.

$$L_t = D_t \forall t \quad (26)$$

where  $L_t$  are nominal loans to domestic firms. The solution to the previous problem is  $i_t^L = i_t^d \forall t$ .

### 2.4 *F*-Economy's Financial Intermediary

Analogously, the *F*-economy has a competitive representative financial intermediary that supplies intra period loans  $L_t^*$  to the representative household. It takes aggregate deposits  $D_t^*$  from local firms and lends them to the *F*-economy representative foreign household. The financial intermediary obtains from firms  $\frac{D_t^*}{1+i_t^{*d}}$  units of *F*-economy currency at the beginning of every period, which in turn allows it to lend to the household  $\frac{L_t^*}{1+i_t^{*L}}$ . At the end of the period, it gets paid from the household  $L_t^*$  and gives back to firms  $D_t^*$ , where  $i_t^{*d}$  is the net nominal interest rate on deposits. Since we are assuming this sector is competitive and there are no intermediation issues we have that  $i_t^{*L} = i_t^{*d}$  and  $L_t^* = D_t^* \forall t$ .

## 2.5 Domestic Firms' Problem

The production block of the economy is a simplified version of that of Mendoza and Yue (2012). The  $H$ -economy is populated by a unit mass of monopolistic competitive firms, indexed by  $i \in [0, 1]$ . Therefore, they take as given the downward sloping demand curves for the good they produce, which could be used as a consumption or as an intermediate good in either economy. Firms require labor and a composite intermediate good to generate output.<sup>9</sup> The technology of  $H$ -economy firms is the same as the one of  $F$ -economy firms.<sup>10</sup>

We assume the intermediate good is the same CES composite of  $H$  and  $F$  inputs as the final composite consumption good. Labor is assumed to be immobile across economies. For simplicity, in this version of the model there is no capital. As usual, a particular firm's objective function consists of maximizing discounted profits, which is linear, and implies that there is no room for precautionary savings in the solution to this problem.

We suppose that firms in the  $H$ -economy can only issue intra period domestic currency denominated loans to the local representative household while they require to pay before production takes place a share of their imported intermediate goods bill in foreign currency (equation (29) and (31)). These loans are supplied and repaid within a period and are not accumulated. Specifically, firms are assumed to borrow  $\frac{L_t(i)}{1+i_t^L}$  from the financial intermediary in order to demand imported inputs. After firms obtain the intra period loan, they exchange the domestically denominated currency loan for foreign currency with the domestic household (through the domestic financial intermediary) which in turn requires domestic currency for consumption purposes but has foreign currency as part of its assets at the beginning of every period. Equation (31) must be satisfied for the firm to have enough foreign currency to meet working capital requirements. Therefore, every firm  $i \in [0, 1]$  in the  $H$ -economy pays in advance to foreign exporting firms foreign money balances equal to  $Z_t^{*f}(i) = \frac{L_t(i)}{(1+i_t^L)e_t} = \int_0^1 \nu P_{F,t}^{m*}(j) m_{F,t}(j) dj$ .

At the end of the period, when goods have been produced and sold, firms pay wages, domestic inputs and the total imported inputs bill. For the latter operation, firm  $i$  gets back its payment in advance in foreign currency plus an intra period interest rate  $i_t^L$ , which totals in foreign currency  $(1+i_t^L)Z_t^{*f}(i)$ .<sup>11</sup> In the same operation, the firm exchanges foreign currency

<sup>9</sup>Mendoza and Yue (2012) also add a time-invariant capital stock as a third factor of production. Perhaps more important for the purpose of the present paper, since they model a SOE in which there is default in equilibrium, they assume there is a subset of imported inputs that do not require any working capital. This allows firms to continue importing such inputs even in the event of a default. In this model, I do not require to model the extensive margin of working capital. Therefore, only the intensive margin plays a role.

<sup>10</sup>The value of the parameters will be allowed to be economy-specific.

<sup>11</sup>Let  $D_t^*(j)$  be an intra period deposit in foreign currency by a foreign firm  $j \in [0, 1]$  in the foreign financial

to the domestic household for domestic one for an amount of  $e_t(1 + i_t^L)Z_t^{*f}(i)$  which from equation (31) is equivalent to  $L_t(i)$ . Hence, this amount is sufficient to repay back the loan to the financial intermediary. On the other hand, the domestic household uses the domestic currency denominated deposit that it gets back at the end of the period to buy the foreign currency to start next period with foreign money balances equal to  $(1 + i_t^d)Z_t^*$ .

We envision a set up in which firms face difficulties in gathering information regarding the terms of borrowing abroad for working capital purposes and therefore, delegate intertemporal substitution of resources to the domestic household. For this purpose and as exposed before, the latter agent has access to a non-contingent bond for international borrowing and lending in the face of economy specific shocks. The fact that households have access to non-contingent bonds implies that consumption risks sharing across economies will be lower as compared to a benchmark of complete markets.

Formally, taking prices  $\left\{ w_t, \left\{ \frac{P_{H,t}^m(i)}{P_t} \right\}_{i \in [0,1]}, \left\{ \frac{e_t P_{F,t}^{m*}(j)}{P_t} \right\}_{j \in [0,1]} \right\}, \{Q_t, e_t, i_t^L\}$  and demand functions  $Y_{c,H,t}(i) \equiv c_{H,t}(i) = (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\eta} c_t$ ,  $Y_{m,H,t}(i) \equiv m_{H,t}(i) = (1 - \omega) \left( \frac{P_{H,t}^m(i)}{P_{M,t}} \right)^{-\eta} M_t$ ,  $Y_{c^*,H,t}(i) \equiv c_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}(i)}{e_t P_t^*} \right)^{-\eta} c_t^*$  and  $Y_{m^*,H,t}(i) \equiv m_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}^m(i)}{e_t P_t^*} \right)^{-\eta} M_t^*$  as given, firm  $i \in [0, 1]$  in the  $H$ -economy solves:

$$\max_{\left\{ L_t(i), \frac{P_{H,t}(i)}{P_t}, \frac{P_{H,t}^m(i)}{P_t}, l_t(i), \{m_{H,t}(i)\}_{i \in [0,1]}, \{m_{F,t}(j)\}_{j \in [0,1]} \right\}} E_t \sum_{s=t}^{\infty} d_{t,s} \frac{\Pi_s(i)}{P_s} \quad (27)$$

s.t.

$$\begin{aligned} \frac{\Pi_t(i)}{P_t} &= \frac{L_t(i)}{(1 + i_t^L)P_t} - \frac{L_t(i)}{P_t} \\ + \mu_H &\left( \frac{P_{H,t}(i)}{P_t} Y_{c,H,t}(i) + \frac{P_{H,t}^m(i)}{P_t} Y_{m,H,t}(i) \right) + \mu_F Q_t \left( \frac{P_{H,t}(i)}{e_t P_t^*} Y_{c^*,H,t}(i) + \frac{P_{H,t}^m(i)}{e_t P_t^*} Y_{m^*,H,t}(i) \right) \\ &- w_t l_t(i) - \int_0^1 \frac{P_{H,t}^m(i)}{P_t} m_{H,t}(i) di - \int_0^1 \frac{e_t P_{F,t}^{m*}(j)}{P_t} m_{F,t}(j) dj, \forall t \end{aligned} \quad (28)$$

intermediary that earns a net nominal interest rate of  $i_t^{*d}$ . From first order conditions of domestic and foreign households we obtain that  $i_t^{*d} = i_{t-1}^* = i_t^L$ . At the end of the period when the foreign firm  $j$  gets back its deposit plus interest rate it obtains  $D_t^*(j) = \frac{\mu_H}{\mu_F} \int_0^1 Z_t^{*f}(i)(1 + i_t^L)di$ . This is exactly the amount of foreign currency the exporting firm  $j$  requires giving back to a particular importing firm  $i \in [0, 1]$ , that we assume is randomly assigned to, which scaled up for economies size totals  $\int_0^1 Z_t^{*f}(i)(1 + i_t^L)di = (1 + i_t^L)Z_t^{*f}(i)$ . See Appendix A.3.

$$\int_0^1 \nu e_t P_{F,t}^{m^*}(j) m_{F,t}(j) dj \leq \frac{L_t(i)}{1 + i_t^L}, \forall t \quad (29)$$

$$\mu_H(Y_{c,H,t}(i) + Y_{m,H,t}(i)) + \mu_F(Y_{c,H,t}^*(i) + Y_{m,H,t}^*(i)) \leq e^{z_t} M_t(i)^{\phi_H} l_t(i)^{1-\phi_H}, \forall t \quad (30)$$

where  $L_t(i)$  must satisfy

$$\frac{L_t(i)}{1 + i_t^L} = e_t Z_t^{*f}(i), \forall t \quad (31)$$

and with

$$M_t(i) = \left( \int_0^1 \left[ (1 - \omega)^{\frac{1}{\eta}} m_{H,t}(i)^{\frac{\eta-1}{\eta}} + \omega^{\frac{1}{\eta}} m_{F,t}(i)^{\frac{\eta-1}{\eta}} \right] di \right)^{\frac{\eta}{\eta-1}}, \eta > 1 \quad (32)$$

and  $z_t = (1 - \rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z$ ,  $\varepsilon_t^z \sim iid N(0, \sigma_{\varepsilon^z})$  with  $|\rho_z| < 1$  and the no-Ponzi condition  $\lim_{s \rightarrow \infty} E_t \left[ d_{t,s} \frac{L_{t+s}(i)}{P_{t+s}} \right] \leq 0$ , where  $d_{t,s}$ ,  $s \geq t$  is the firm's discount factor which equals  $\beta^{s-t} E_t \left[ \left( \frac{c_s}{c_t} \right)^{-\sigma} \right]$ , given that the  $H$ -economy representative household is assumed to own the firms.<sup>12</sup> The parameter  $\nu \in [0, 1]$  measures the share of the imported inputs bill that must be paid in advance,  $Q_t$  is the real exchange rate (defined as  $Q_t \equiv \frac{e_t P_t^*}{P_t}$ ),  $Y_t(i)$  is total output of the firm,<sup>13</sup>  $M_t(i)$  is the composite intermediate good used for production,  $m_{H,t}(i)$  is the domestic input of type  $i \in [0, 1]$  demanded by the firm,  $m_{F,t}(j)$  is the imported input of type  $j \in [0, 1]$  used in production,  $P_{H,t}^m(i)$  is the price of intermediate good  $H$  of type  $i \in [0, 1]$ ,  $P_{F,t}^m(j)$  is the price of intermediate good  $F$  of type  $j \in [0, 1]$  and  $\phi_H \in (0, 1)$  is the share of intermediate inputs in the production function of the  $H$ -economy. Moreover,  $\frac{\Pi_t(i)}{P_t}$  are real profits of the firm  $i$  at period  $t$ ,  $L_t(i) \geq 0$  are intra period nominal loans denominated in domestic currency and  $Z_t^{*f}(i)$  is the foreign currency demand of the domestic firm  $i$ . The working capital intensity increases in the parameter  $\nu$  and it is considered exogenous in the model.

It is important to notice that any kind of payment in advance intermediated by banks like

---

<sup>12</sup>Notice that we are assuming that the variance-covariance matrix of  $\begin{pmatrix} \varepsilon_t^z \\ \varepsilon_t^{z^*} \end{pmatrix}$  is equal to  $\begin{pmatrix} \sigma_{\varepsilon^z}^2 & 0 \\ 0 & \sigma_{\varepsilon^{z^*}}^2 \end{pmatrix}$ , where  $\varepsilon_t^{z^*}$  is the productivity shock of the  $F$ -economy. See Appendix A.3.

<sup>13</sup> $\mu_H(Y_{c,H,t}(i) + Y_{m,H,t}(i)) + \mu_F(Y_{c,H,t}^*(i) + Y_{m,H,t}^*(i))$  holds every period, where  $Y_{c,H,t}(i)$  is the production of  $c_{H,t}(i)$ ,  $Y_{m,H,t}(i)$  the production of  $m_{H,t}(i)$ ,  $Y_{c,H,t}^*(i)$  the production of good  $c_{H,t}^*(i)$  and  $Y_{m,H,t}^*(i)$  the one of  $m_{H,t}^*(i)$ , where  $m_{H,t}^*(i)$  is the demand for the  $H$ -economy good by the foreign economy used as an intermediate good.

those referred by Antràs and Foley (2015) such as: i) a wire transfer in advance, ii) 20% deposit and 80% wire transfer in advance, or iii) a 10% wire transfer in advance and 90% prior to arrival and so on, they all perfectly fit in this framework. The key friction we model is therefore the lack of credit in foreign currency from the  $H$ -economy firm's perspective (perhaps, due to imperfect enforceability of contracts), and hence, the need to fund working capital requirements in foreign currency for imported inputs in the local market. However, it must be highlighted that from the perspective of the financial intermediary the borrowing and lending that takes place in this model on an intra period basis is frictionless in the sense that there is nothing in the environment that makes the rate on intra period loans to be different to that of intra period deposits.

From the first order conditions of problem (27) we obtain (see derivation in Appendix A.2):

$$\frac{P_{H,t}(i)}{P_t} = \frac{P_{H,t}^m(i)}{P_t} = \frac{P_{H,t}}{P_t} \quad \forall i \in [0, 1] \quad (33)$$

where the latter equality follows from the fact that all firms  $i \in [0, 1]$  in the  $H$ -economy are identical. Moreover,

$$w_t = \left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} (1 - \phi_H) e^{z_t} M_t(i)^{\phi_H} l_t(i)^{-\phi_H} \quad (34)$$

Define  $T_t \equiv \frac{e_t P_{F,t}^*}{P_{H,t}}$  as the terms of trade and  $wk_t \equiv \nu i_t^L$  as the working capital cost. Then, we also obtain

$$T_t (1 + wk_t) = \frac{\omega^{\frac{1}{\eta}} m_{F,t}^{-\frac{1}{\eta}}(i)}{(1 - \omega)^{\frac{1}{\eta}} m_{H,t}^{-\frac{1}{\eta}}(i)} \quad (35)$$

Given that the firm  $i$  faces the same prices for their inputs,  $m_{H,t}(i) = m_{H,t}$  and  $m_{F,t}(i) = m_{F,t}$ , the above condition can therefore be expressed as

$$\frac{m_{F,t}}{m_{H,t}} = \frac{\omega}{1 - \omega} (T_t (1 + wk_t))^{-\eta} \quad (36)$$

where we can see the partial equilibrium link between variations in working capital cost, the intratemporal elasticity of substitution and the relative demand for inputs  $\frac{m_{F,t}}{m_{H,t}}$ .<sup>14</sup> Observe that this formulation of working capital is isomorphic to the Samuelson (1954) formulation of ice-

<sup>14</sup>An analogous relationship can be obtained for the  $F$ -economy, except that in such a case there is no working



berg trade costs. This point has been raised before by Schmidt-Eisenlohr (2013).<sup>15</sup> Moreover, as anticipated working capital requirements on the imported input drive a wedge between the marginal rate of transformation of the foreign and the domestic intermediate good and the terms of trade.<sup>16</sup> Through the lens of the model that we present here and given the evidence presented in Antràs and Foley (2015) this “wedge” may also be thought as arising from a lack of commitment problem in some international transactions which is solved (in contractual terms) by a payment in advance constraint.

Manipulating  $H$ -economy firm first order conditions it is possible to obtain

$$\begin{aligned} & \phi_H \left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} \\ &= \frac{M_t(i)}{Y_t(i)} \left( \int_0^1 \left[ (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{1-\eta} + \omega \left( (1 + wk_t) \frac{e_t P_{F,t}^*(i)}{P_t} \right)^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}} \end{aligned} \quad (37)$$

and therefore,

$$\phi_H \left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} = \frac{M_t(i)}{Y_t(i)} \frac{P_{M,t}}{P_t} \quad (38)$$

where we define  $P_{M,t}$  as the ideal price index of intermediate goods in the  $H$ -economy net of working capital costs as

$$P_{M,t} \equiv \left( \int_0^1 [(1 - \omega) P_{H,t}(i)^{1-\eta} + \omega ((1 + wk_t) e_t P_{F,t}^*(i))^{1-\eta}] di \right)^{\frac{1}{1-\eta}} \quad (39)$$

which implies that the intermediate goods ideal price index in the  $H$ -economy is affected by the working capital cost through the (net) cost of foreign inputs, as it can be seen from the capital cost term:

$$\frac{m_{F,t}^*}{m_{H,t}^*} = \frac{1 - \omega^*}{\omega^*} T_t^{-\eta}$$

where  $\omega^*$  is the home bias parameter in the CES aggregator of the  $F$ -economy. See Appendix A.3 for the derivation of  $F$ -economy firms first order conditions.

<sup>15</sup>Schmidt-Eisenlohr (2013) trade finance formulation contract depends on the interest rates of the source and destination country and the probability of enforcing contracts in each of them. Although richer in terms of the contracts he considers, namely, cash in advance, open account and letter of credit, he does not attempt to provide a general equilibrium model but rather a characterization of each of the contracts and the environment in which they evolve endogenously. The model of Antràs and Foley (2015) is also a partial equilibrium model that explain the differences in how trade is financed depending on contractual enforcement. That model, among other things, incorporates some dynamics into the one of Schmidt-Eisenlohr (2013).

<sup>16</sup>In the framework of Meza, Pratap, and Urrutia (2019) this friction exacerbates a static misallocation of inputs that leads firms to pay a higher interest rates (with respect to the ones that would prevail in the absence of the friction) leading, by definition, to an inefficient allocation of inputs (see Chari, Kehoe, and McGrattan (2007)).

last term of the previous equation. Thus, variations in the working capital cost, either positive or negative, may have an impact on the  $H$ -firm demand for the domestic and the imported inputs. In equilibrium, variations in the working capital cost have an effect on the mix of inputs in the  $H$ -economy. As it will be seen latter, it also has effects on the composition of inputs in the  $F$ -economy firm through general equilibrium effects.

Notice that we can normalize the above equation by  $P_t$  to obtain

$$\frac{P_{M,t}}{P_t} = \left( \int_0^1 \left[ (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{1-\eta} + \omega \left( (1 + wk_t) \frac{e_t P_{F,t}^*(i)}{P_t} \right)^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}} \quad (40)$$

Since  $1 = \left( \int_0^1 \left[ (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{1-\eta} + \omega \left( \frac{e_t P_{F,t}^*(i)}{P_t} \right)^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}}$ , we can re-express equation (40) as

$$\frac{P_{M,t}}{P_t} = \frac{\left( \int_0^1 \left[ (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{1-\eta} + \omega \left( (1 + wk_t) \frac{e_t P_{F,t}^*(i)}{P_t} \right)^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}}}{\left( \int_0^1 \left[ (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{1-\eta} + \omega \left( \frac{e_t P_{F,t}^*(i)}{P_t} \right)^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}}} \quad (41)$$

and therefore, from equation (41) it is clear that if  $wk_t > 0$ , then  $\frac{P_{M,t}}{P_t} > 1$ . When  $wk_t = 0$ , then  $\frac{P_{M,t}}{P_t} = 1$ .

Also, observe that from equations (34) and (38), it follows that

$$\left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} Y_t(i) = \frac{P_{M,t}}{P_t} M_t(i) + w_t l_t(i) = mc_t Y_t(i) \quad (42)$$

where  $mc_t = e^{-zt} \left( \frac{w_t}{1 - \phi_H} \right)^{1 - \phi_H} \left( \frac{P_{M,t}}{P_t \phi_H} \right)^{\phi_H}$ . Given that firms in the  $H$ -economy are identical, it follows that  $P_{H,t}(i) = P_{H,t} \forall i \in [0, 1]$ . Notice that the previous condition implies that

$$\left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}}{P_t} = e^{-zt} \left( \frac{w_t}{1 - \phi_H} \right)^{1 - \phi_H} \left( \frac{P_{M,t}}{P_t \phi_H} \right)^{\phi_H} \quad (43)$$

which means that the price of good  $H$  in real terms is a mark-up of its real marginal cost. Similar conditions hold for the  $F$ -economy firm (see Appendix A.3.) This result is a direct consequence of the monopolistic competition structure in which firms operate and as it will be seen below, gives rise to positive profits in equilibrium. Notice that the marginal cost for  $H$ -economy firms is a function of  $P_{M,t}$ , which in turn is a function of the working capital cost.

Moreover, dividing the analogous expression of (43) for the  $F$ -economy by (43) we obtain

$$\frac{T_t}{Q_t} = \frac{e^{-z_t^*} \left( \frac{w_t^*}{1-\phi_F} \right)^{1-\phi_F} \left( \frac{1}{\phi_F} \right)^{\phi_F}}{e^{-z_t} \left( \frac{w_t}{1-\phi_H} \right)^{1-\phi_H} \left( \frac{P_{M,t}}{P_t \phi_H} \right)^{\phi_H}} \quad (44)$$

## 2.6 Demand for Intermediate Goods

To formalize the procedure to obtain the demand for intermediate goods in the  $H$ -economy, we can set up the following problem. Given  $\left\{ \frac{P_{H,t}^m(i)}{P_t} \right\}_{i \in [0,1]}$  and  $\left\{ \frac{(1+wk_t)e_t P_{F,t}^{m^*}(i)}{P_t} \right\}_{i \in [0,1]}$

$$\min_{\{m_{H,t}(i)\}_{i \in [0,1]}, \{m_{F,t}(i)\}_{i \in [0,1]}} \int_0^1 \left[ \frac{P_{H,t}^m(i)}{P_t} m_{H,t}(i) + \frac{(1+wk_t)e_t P_{F,t}^{m^*}(i)}{P_t} m_{F,t}(i) \right] di \quad (45)$$

s.t.

$$\left( \int_0^1 \left[ (1-\omega)^{\frac{1}{\eta}} m_{H,t}^{\frac{\eta-1}{\eta}}(i) + \omega^{\frac{1}{\eta}} m_{F,t}^{\frac{\eta-1}{\eta}}(i) \right] di \right)^{\frac{\eta}{\eta-1}} = 1 \quad (46)$$

Let  $\tilde{\Theta}_t$  be the Lagrange multiplier of the constraint. The first order conditions of the problem are

$$\frac{P_{H,t}^m(i)}{P_t} = \tilde{\Theta}_t M_t^{\frac{1}{\eta}} (1-\omega)^{\frac{1}{\eta}} m_{H,t}(i)^{-\frac{1}{\eta}} \quad \forall i \in [0,1] \quad (47)$$

$$\frac{(1+wk_t)e_t P_{F,t}^{m^*}(i)}{P_t} = \tilde{\Theta}_t M_t^{\frac{1}{\eta}} \omega^{\frac{1}{\eta}} m_{F,t}(i)^{-\frac{1}{\eta}} \quad \forall i \in [0,1] \quad (48)$$

Raising (47) and (48) to the power of  $1-\eta$ , we obtain

$$(1-\omega) \left( \frac{P_{H,t}^m(i)}{P_t} \right)^{1-\eta} = \tilde{\Theta}_t^{1-\eta} M_t^{\frac{1-\eta}{\eta}} (1-\omega)^{\frac{1}{\eta}} m_{H,t}(i)^{\frac{\eta-1}{\eta}} \quad \forall i \in [0,1] \quad (49)$$

$$\omega \left( \frac{(1+wk_t)e_t P_{F,t}^{m^*}(i)}{P_t} \right)^{1-\eta} = \tilde{\Theta}_t^{1-\eta} M_t^{\frac{1-\eta}{\eta}} \omega^{\frac{1}{\eta}} m_{F,t}(i)^{\frac{\eta-1}{\eta}} \quad \forall i \in [0,1] \quad (50)$$

Adding up (49) and (50) and integrating over  $i$  we get

$$\int_0^1 \left[ (1-\omega) \left( \frac{P_{H,t}^m(i)}{P_t} \right)^{1-\eta} + \omega \left( \frac{(1+wk_t)e_t P_{F,t}^{m^*}(i)}{P_t} \right)^{1-\eta} \right] di = \tilde{\Theta}_t^{1-\eta} \quad (51)$$

Hence,

$$\tilde{\Theta}_t = \left( \int_0^1 \left[ (1 - \omega) \left( \frac{P_{H,t}^m(i)}{P_t} \right)^{1-\eta} + \omega \left( \frac{(1 + wk_t)e_t P_{F,t}^{m^*}(i)}{P_t} \right)^{1-\eta} \right] di \right)^{\frac{1}{1-\eta}} \equiv \frac{P_{M,t}}{P_t} \quad (52)$$

Plugging (52) into (47) and (48) we obtain the following demand functions

$$m_{H,t}(i) = (1 - \omega) \left( \frac{P_{H,t}^m(i)}{P_{M,t}} \right)^{-\eta} M_t \quad \forall i \in [0, 1] \quad (53)$$

$$m_{F,t}(j) = \omega \left( \frac{(1 + wk_t)e_t P_{F,t}^{m^*}(j)}{P_{M,t}} \right)^{-\eta} M_t \quad \forall j \in [0, 1] \quad (54)$$

Similarly, the corresponding demand functions for the  $F$ -economy are

$$m_{F,t}^*(j) = (1 - \omega^*) \left( \frac{P_{F,t}^{m^*}(j)}{P_t^*} \right)^{-\eta} M_t^* \quad \forall j \in [0, 1] \quad (55)$$

$$m_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}^m(i)}{e_t P_t^*} \right)^{-\eta} M_t^* \quad \forall i \in [0, 1] \quad (56)$$

where  $m_{F,t}^*(j)$  is the demand for the  $F$ -economy good  $j \in [0, 1]$  by the foreign economy as an input while  $m_{H,t}^*(i)$  is the one for the  $H$ -economy good  $i \in [0, 1]$ . Moreover,  $M_t^*$  is the intermediate composite good in the foreign economy.

## 2.7 Firms' Profits

Real profits for firm  $i \in [0, 1]$  in the  $H$ -economy can be expressed as

$$\frac{\Pi_t(i)}{P_t} = \frac{P_{H,t}(i)}{P_t} Y_t(i) - \int_0^1 \frac{P_{H,t}^m(i)}{P_t} m_{H,t}(i) di - \int_0^1 \frac{e_t P_{F,t}^{m^*}(j)}{P_t} m_{F,t}(j) dj - w_t l_t(i) - \frac{i_t^L}{1 + i_t^L} \frac{L_t(i)}{P_t} \quad (57)$$

and since  $\frac{L_t(i)}{1 + i_t^L} = \int_0^1 \nu e_t P_{F,t}^{m^*}(j) m_{F,t}(j) dj$  we obtain

$$\frac{\Pi_t(i)}{P_t} = \frac{P_{H,t}(i)}{P_t} Y_t(i) - \int_0^1 \frac{P_{H,t}^m(i)}{P_t} m_{H,t}(i) di - (1 + \nu i_t^L) \int_0^1 \left( \frac{e_t P_{F,t}^{m^*}(j)}{P_t} m_{F,t}(j) \right) dj - w_t l_t(i) \quad (58)$$

where the term  $\nu i_t^L \int_0^1 \frac{e_t P_{F,t}^{m^*}(j)}{P_t} m_{F,t}(j) dj$  represents the financial cost of the working capital constraint for imported inputs. Therefore, using the relative price  $\frac{P_{M,t}}{P_t}$  in expression (40), real profits for firm  $i \in [0, 1]$  in the  $H$ -economy can be expressed as

$$\frac{\Pi_t(i)}{P_t} = \frac{P_{H,t}(i)}{P_t} Y_t(i) - \frac{P_{M,t}}{P_t} M_t(i) - w_t l_t(i) = \frac{P_{H,t}(i)}{P_t} \frac{Y_t(i)}{\eta} \quad (59)$$

where the latter equality follows from expression (42).

On the other hand, profits of the firm  $j \in [0, 1]$  in the  $F$ -economy firm take the form

$$\begin{aligned} \frac{\Pi_t^*(j)}{P_t^*} &= \frac{P_{F,t}^*(j)}{P_t^*} Y_{F,t}^*(j) - \int_0^1 \frac{P_{H,t}^m(i)}{e_t P_t^*} m_{H,t}^*(i) di \\ &- \int_0^1 \frac{P_{F,t}^{m^*}(j)}{P_t^*} m_{F,t}^*(j) dj - w_t^* l_t^*(j) + \frac{i_t^{*d}}{1 + i_t^{*d}} \frac{D_t^*(j)}{P_t^*} \end{aligned} \quad (60)$$

Finally, it can be shown (see Appendix A.4, (expression A.54)) that substituting the expression for aggregate domestic profits, equation (A.53), into the domestic budget constraint, equation (A.49) and imposing the  $H$ -economy government budget constraint, equation (A.50), one obtains

$$c_t + M_t + \frac{e_t(Z_{t+1}^* - Z_t^*)}{P_t} + \frac{e_t B_t^*}{(1 + i_t^*)P_t} = \frac{e_t B_{t-1}^*}{P_t} + \frac{P_{H,t}}{P_t} Y_t \quad (61)$$

On the other hand, (see Appendix A.4) plugging aggregate foreign firms' profits (A.57) into the foreign household budget (A.56), we obtain (see Appendix A.4, expression (A.58))

$$c_t^* + M_t^* + \frac{(\tilde{Z}_{t+1}^* - \tilde{Z}_t^*)}{P_t^*} + \frac{\tilde{B}_t^*}{(1 + i_t^*)P_t^*} = \frac{\tilde{B}_{t-1}^*}{P_t^*} + \frac{P_{F,t}^*}{P_t^*} Y_t^* \quad (62)$$

Finally, a condition to close the model is that

$$\frac{e_t B_t^*}{(1 + i_t^*)P_t} - \frac{e_t B_{t-1}^*}{P_t} = \frac{\mu_F}{\mu_H} \frac{P_{H,t}}{P_t} (c_{H,t}^* + m_{H,t}^*) - \frac{e_t P_{F,t}^*}{P_t} (c_{F,t} + (1 + w k_t) m_{F,t}) \quad (63)$$

where the right-hand side of the previous equation is the trade balance of the  $H$ -economy minus the term  $\frac{w k_t e_t P_{F,t}^* m_{F,t}}{P_t}$ .

## 2.8 Monetary Policy Rules

Monetary policy in the foreign economy is described by a conventional Taylor rule<sup>17</sup>

$$i_t^* = \rho_{i^*} i_{t-1}^* + \beta_{\pi^*} \tilde{\pi}_t^* + \beta_{Y^*} \tilde{Y}_t^* + \varepsilon_t^{i^*} \quad (64)$$

where  $\tilde{\pi}_t^*$  is the foreign economy inflation gap,  $\tilde{Y}_t^*$  is the foreign output gap and  $\varepsilon_t^{i^*}$  is a foreign unexpected monetary shock. Analogously, monetary policy in the home economy is represented by a conventional Taylor rule of the form

$$i_t = \rho_i i_{t-1} + \beta_{\pi} \tilde{\pi}_t + \beta_Y \tilde{Y}_t + \varepsilon_t^i \quad (65)$$

where  $\tilde{\pi}_t$  is the inflation gap,  $\tilde{Y}_t$  is the output gap and  $\varepsilon_t^i$  is an unexpected monetary shock. Following Taylor (1993), we set  $\beta_{\pi^*} = \beta_{\pi} = 1.5$  and  $\beta_{Y^*} = \beta_Y = 0.5$ . We also set for illustrative purposes the persistence parameter  $\rho_{i^*} = \rho_i = 0.66$  in line with that of Clarida, Gali, and Gertler (1999).

## 2.9 Market Clearing Conditions

We must provide equilibrium conditions for product, assets and labor markets. We must also specify intertemporal budget constraints in equilibrium for both economies. Let  $\mu_H$  and  $\mu_F$  be the share of the population at home and foreign economy, respectively. Equilibrium in  $H$  and  $F$  goods markets, respectively, requires

$$\begin{aligned} \int_0^1 Y_t(i) di &= \mu_H \int_0^1 Y_{H,t}(i) di + \mu_F \int_0^1 Y_{H,t}^*(i) di \\ &= \mu_H \left( \int_0^1 c_{H,t}(i) di + \int_0^1 m_{H,t}(i) di \right) + \mu_F \left( \int_0^1 c_{H,t}^*(i) di + \int_0^1 m_{H,t}^*(i) di \right) \quad \forall t \quad (66) \end{aligned}$$

---

<sup>17</sup>Bernanke (2015) suggests that the simplicity of a Taylor rule should not be interpreted as monetary policy being made automatic but rather systematic. He adds: “The simplicity of the Taylor rule disguises the complexity of underlying judgments that FOMC members must continually make if they are to make a good policy decision.”

$$\begin{aligned}
& \int_0^1 Y_t^*(j) dj = \mu_H \int_0^1 Y_{F,t}(j) dj + \mu_F \int_0^1 Y_{F,t}^*(j) dj \\
& = \mu_H \left( \int_0^1 c_{F,t}(j) dj + \int_0^1 m_{F,t}(j) dj \right) + \mu_F \left( \int_0^1 c_{F,t}^*(j) dj + \int_0^1 m_{F,t}^*(j) dj \right) \quad \forall t \quad (67)
\end{aligned}$$

Equilibrium in asset markets calls for

$$B_t = 0 \quad \forall t \quad (68)$$

$$D_t = \int_0^1 L_t(i) di = \int_0^1 \int_0^1 (1 + i_t^L) \nu e_t P_{F,t}^{m^*}(j) m_{F,t}(j) dj di \quad \forall t \quad (69)$$

$$Z_t^* = \int_0^1 Z_t^{*f}(i) di = \int_0^1 \frac{L_t(i)}{e_t(1 + i_t^L)} di \quad \forall t \quad (70)$$

$$L_t^* = \int_0^1 D_t^*(j) dj = (1 + i_t^d) \frac{\mu_H}{\mu_F} \int_0^1 Z_t^{*f}(i) di \quad \forall t \quad (71)$$

$$M_t^s = Z_t \quad \forall t \quad (72)$$

$$M_t^{*s} = \mu_H Z_t^* + \mu_F \tilde{Z}_t^* \quad \forall t \quad (73)$$

$$\mu_H B_t^* + \mu_F \tilde{B}_t^* = 0 \quad \forall t \quad (74)$$

Equation (68) states that the domestic bond market is inactive in equilibrium. However, equation (69) indicates that local borrowing and lending takes place on an intra period basis. This means that the domestic household deposits must equal local firms' loans. Both of them, in turn, are equal to the value of working capital loans for imported inputs expressed in domestic currency. Equation (70) refers to the equilibrium condition in the foreign currency market within the domestic economy. It states that the foreign currency demanded by local firms to meet working capital requirements for imported inputs must equal the amount of foreign currency the  $H$ -economy representative household has every period.<sup>18</sup> Condition (71) refers to the equilibrium condition in the intra period fund market in the foreign economy.

---

<sup>18</sup>Notice that this equilibrium condition implies that equation (9) collapses to  $P_t c_t = Z_t$ .

Condition (72) equates domestic money supply,  $M_t^s$ , with local money demand while equation (73) is the equilibrium condition in the foreign money market, where  $M_t^{*s}$  is the foreign money supply. Equation (74) indicates that the foreign bond market is the one through which the borrowing and lending across economies occur to smooth economy-specific productivity shocks, albeit imperfectly as compared to a complete markets benchmark.<sup>19</sup>

Equilibrium in home labor market is satisfied when  $l_t$  meets simultaneously supply represented by (8) and the demand as in equation (34). For the case of the foreign labor market,  $l_t^*$  must satisfy analogous equations.

Additionally, the  $F$ -economy government satisfies its budget constraint

$$\frac{M_{t+1}^{*s} - M_t^{*s}}{P_t^*} + \mu_H \left( \frac{B_t^*}{(1 + i_t^*)P_t^*} - \frac{B_{t-1}^*}{P_t^*} \right) + \mu_F \left( \frac{\tilde{B}_t^*}{(1 + i_t^*)P_t^*} - \frac{\tilde{B}_{t-1}^*}{P_t^*} \right) = 0 \quad (75)$$

The analogous condition for the  $H$ -economy government is

$$\frac{M_{t+1}^s - M_t^s}{P_t} + \frac{B_t}{(1 + i_t)P_t} - \frac{B_{t-1}}{P_t} = 0 \quad (76)$$

Using domestic and foreign households' budget constraints and combining it with expressions for firms' profits (equations (57), (60)), (75) and (76) we obtain the feasibility constraint of the two-country world economy which is satisfied by Walras' Law (see Appendix A.4):

$$\mu_H(c_t + M_t) + \mu_F Q_t (c_t^* + M_t^*) = \mu_H \frac{P_{H,t}}{P_t} Y_t + \mu_F Q_t \frac{P_{F,t}^*}{P_t^*} Y_t^* \quad (77)$$

---

<sup>19</sup>The degree to which incomplete financial markets matter for effective consumption risk sharing depends, among other things, on the stochastic processes followed by total factor productivity (TFP) of the modeled economies as shown by Baxter and Crucini (1995) in a two country model with a single consumption good. In particular, they pose a case in which shocks to TFP are permanent (that is, the autoregressive parameter of each economy TFP is one), the spillover coefficient is zero and TFP innovations are correlated. They obtain for that case high output correlations and low consumption correlation across economies. Conversely, when productivity shocks are not permanent, equilibrium allocations of economies that only trade non-contingent bonds may be close to the complete markets equilibrium allocations. These results crucially rely on being a single good the one that is traded and consumed. Cole and Obstfeld (1991) model a two country economy with two goods under incomplete markets where each country is endowed by a single good which follows a stochastic process. They highlight the role of adjustments in terms of trade as a mechanism for pooling output risks independently of trade in assets since the benefits of country-specific gains propagate via prices to the other economy, regardless of technological spillovers. Conversely, Corsetti, Dedola, and Leduc (2008) point to a lower consumption risk sharing via terms of trade in an incomplete markets setting due to wealth effects after a productivity shock that drive demand up, crowding out external demand for goods. They obtain this result for low values of the intratemporal elasticity of substitution and also for the case of relatively high values of said elasticity and highly persistent exogenous shocks, among them, TFP ones, although other shocks are also considered such as preferences' ones.



**Table 1: List of Parameters**

Parameter	Description	Value
$\beta$	Discount factor	0.99
$\sigma$	Relative risk aversion	2
$1/\chi$	$H$ -economy Frisch elasticity of labor supply	3.27
$1/\chi^*$	$F$ -economy Frisch elasticity of labor supply	1
$\varphi$	Leisure share preference parameter in $H$ -economy	1.48
$\varphi^*$	Leisure share preference parameter in $F$ -economy	3
$\omega$	$H$ -economy home bias parameter	0.31
$\omega^*$	$F$ -economy home bias parameter	0.07
$\eta$	Intratemporal elasticity of substitution	4
$\phi$	Intermediate goods share in production in $H$ -economy	0.525
$\phi^*$	Intermediate goods share in production in $F$ -economy	0.50
$\mu_H$	Relative size of $H$ -economy in terms of labor force, 2000-2019	0.23
$\mu_F$	Relative size of $F$ -economy in terms of labor force, 2000-2019	0.77
$\nu$	Share of imported intermediate goods bill paid in advance	0.42
$\rho_z$	Autoregressive coefficient of productivity in $H$ -economy	0.92
$\rho_z^*$	Autoregressive coefficient of productivity in $F$ -economy	0.97

*Definition.* An equilibrium in this economy is contingent plans for allocations  $\{c_t, c_{H,t}, c_{F,t}, c_t^*, c_{H,t}^*, c_{F,t}^*, l_t, l_t^*, B_t, B_t^*, \tilde{B}_t^*, D_t, L_t^*, Z_{t+1}, Z_{t+1}^*, \tilde{Z}_{t+1}^*, M_t^s, M_t^{*s}\}$ , production plans  $\{M_t(i), m_{H,t}(i), m_{F,t}(i), l_t(i), Y_t(i), L_t(i), \Pi_t(i), Z_{t+1}^{*f}(i)\}_{i \in [0,1]}$ ,  $\{M_t^*(j), m_{H,t}^*(j), m_{F,t}^*(j), l_t^*(j), Y_t^*(j), \Pi_t^*(j), D_t^*(j)\}_{j \in [0,1]}$  and prices  $\{P_t, P_t^*, P_{M,t}, \{P_{H,t}(i)\}_{i \in [0,1]}, \{P_{F,t}^*(j)\}_{j \in [0,1]}, \{P_{H,t}^m(i)\}_{i \in [0,1]}, \{P_{F,t}^{*m}(j)\}_{j \in [0,1]}, e_t, Q_t, T_t, i_t, i_t^*, i_t^d, i_t^L, i_t^{*d}, i_t^{*L}, W_t, W_t^*\}$  such that

- i)  $H$  and  $F$  economy representative consumers solve their optimization problems,
- ii)  $H$  and  $F$  economy firms solve their maximizations problems,
- iii) Monetary authorities follow monetary rules, and
- iv) Markets clear.

### 3 Numerical Exercise

In this section we perform a numerical exercise to illustrate the functioning of the model. The parameters of the model are presented in Table 1.

The discount factor and the relative risk aversion parameters are standard in the literature and are assumed to be equal across economies. Labor market parameters are different across

economies. In particular, the Frisch elasticity of labor supply for the  $H$ -economy is the one in Leyva and Urrutia (2020) for Mexico. This elasticity takes the value of 3.27. The Frisch elasticity of the U.S. is as in Itskhoki and Mukhin (2021) and close to some micro estimates as found by Dyrda, Kaplan, and Ríos-Rull (2012). The leisure share parameter  $\varphi$  is also set as in Leyva and Urrutia (2020) and for the U.S. I consider the standard parameter of 3 in line with Chari, Kehoe, and McGrattan (2000).

The home bias parameter for Mexico is calculated as the imports of goods and services as a share of GDP from 2000 to 2Q2020 using data from National Accounts by INEGI. The resulting parameter is 0.31. The home bias parameter for the U.S. is set at 0.07 as in Itskhoki and Mukhin (2021). The elasticity of substitution between domestic and foreign goods, which is assumed to be the same across economies takes the value of 4, very close to the value in Bernard, Eaton, Jensen, and Kortum (2003). Nonetheless, I am aware that recent work by Rothert (2020) points toward a considerably lower value of this parameter.

The parameter  $\phi_H$  takes the value of 0.525 based on Chilean plant-level data as in Ramnarayanan (2017). For the case of the U.S., the parameter  $\phi_F$  takes the value of 0.50 as in Itskhoki and Mukhin (2017) and Jones (2011). The parameters  $\mu_H$  and  $\mu_F$  that represent the share of the labor force of Mexico and the U.S. as a total are set at 0.23 and 0.77, respectively, which is the average of such a figure using labor force data from 2000 to 2019 from the World Bank.

The key parameter of the model is the one which captures the share of the imported intermediate goods bill that must be paid in advance for firms in the  $H$ -economy. As a baseline case, we will consider a value of  $\nu = 0.42$  which we proxy by the *share of sales* in cash in advance terms in the study of Antràs and Foley (2015). Those sales correspond to a single U.S. based firm that exports frozen and refrigerated food products, primarily poultry to more than 140 countries over the 1996-2009 period. As stressed out by Antràs and Foley (2015) the cash in advance terms typically involve a wire transfer or deposit in advance of shipping goods. Hence, although these payments are intermediated by banks, a financing relationship between a financial institution and a firm does not exist. Antràs and Foley (2015) document many cash in advance terms. All these terms imply that before the import arrives to its destination, 100% must have been covered either as a wire transfer and/or a deposit. So, for the purpose of our numerical exercise, we will simply take as given the value of  $\nu$  and ignore intra period payments. It is also important to keep in mind that the authors find that this payment category (with respect to foreign counterparts) is higher in countries with civil law, low enforceability of contracts and payment delays. Those features are likely to be present in most emerging economies. Nonetheless, they also find that among other factors, as the relationship

between exporting and importing firms evolves over time, parties involved tend to rely more on credit.<sup>20</sup>

We set the autoregressive parameter of the productivity process in Mexico,  $\rho_z$  at 0.92 also in line with Leyva and Urrutia (2020) and very close to the values of Aguiar and Gopinath (2007) and Ramanarayanan (2017). For the U.S. the parameter  $\rho_z^*$  is set at 0.97 following Heathcote and Perri (2002) who update the work of Backus, Kehoe, and Kydland (1995). For simplicity, we also assume a spillover coefficient of zero and leave for future work an estimation of the joint productivity process for the two economies. For ease of exposition, I have not made explicit the introduction of small quadratic adjustment costs payable in terms of the good  $F$  for  $B_t^*$  and  $\tilde{B}_t^*$ . I introduce them to get rid of the unit root in the solution of the model.

To illustrate the functioning of the model, Figure 1 shows the response functions of some variables of the model to an unanticipated positive shock  $\epsilon_t^z$  to the productivity of the domestic economy which increases domestic output by 1% immediately. We present two cases. The first one corresponds to the baseline parameterization in which  $\nu = 0.42$  (solid line). For comparison purposes we also set  $\nu = 0.64$  (dashed line), that is, payment in advance requirements for imported inputs are higher.<sup>21</sup>

This figure shows that in both cases the shock is followed by a transitory appreciation of terms of trade and a sudden increase in the interest rate on intra period loans followed by a prolonged temporary decrease. The latter interest rate is one period ahead of the domestic interest rate, as was derived analytically. Notice that when  $\nu = 0.42$  (solid line) the increase in the working capital cost (which is a function of the intra period loans rate, as mentioned before) and its later decline is milder as compared to the case in which  $\nu = 0.64$  (dashed line). Consequently, this leads to inputs recomposition in firms of the domestic economy, which at the margin demand more imported inputs vis-à-vis domestic ones when working capital requirements are higher. Due to general equilibrium effects allocations in the  $F$ -economy are also affected.

Regarding the loan in domestic currency to make the payments in advance in foreign currency,  $L_t$ , it exhibits an increase at the time of the shock and diminishes gradually. Although

---

<sup>20</sup>Using a completely different data set, namely, a disaggregated data set of more than 50,000 export flows from July 2007 to June 2009 that matches Peruvian exports by firm and product to multiple destinations with local bank loans, Paravisini, Rappoport, Schnabl, and Wolfenzon (2015) find that 0.422 of importers pay in advance. When authors condition for loans issued to exporters with positive bank debt and more than one banking relationship, this share is as high as 0.615.

<sup>21</sup>This value is close to the upper bound for the values in Paravisini, Rappoport, Schnabl, and Wolfenzon (2015).

the volume of imported inputs increases temporarily after the shock, the appreciation of terms of trade driven mainly by a fall in the relative price of the good  $F$  vis-à-vis the good  $H$  seems to imply that the combined effect is a transitory reduction in domestic intra period loans with an eventual recovery. The same pattern is presented by foreign intra period deposits (expressed in domestic currency) since they are basically a downsized version of the intra period loans/deposits in the domestic economy.

## 4 Conclusions

I have presented a two-country model with fully flexible prices, no capital accumulation and incomplete markets. The working capital cost for imported inputs in the domestic economy is a function of the domestic nominal interest rate on intra period loans. In this setting, local firms are required to pay in advance a share of their imported inputs bill, which is denominated in foreign currency. However, since we only allow firms to finance this requirement in local currency, this gives rise to a working capital cost that depends on the referred nominal interest rate times the share of imported inputs bill that must be paid in advance. Changes in the working capital cost generate a wedge between local and imported inputs in the domestic economy, which induce compositional effects in intermediate goods at the face of productivity shocks.

I have assumed that firms of one of the economies (the emerging one) are subject to a working capital constraint in foreign currency while firms in the other economy (the advanced one) are not subject to this sort of constraint. Although more research is necessary to fully understand the conditions under which this type of setting emerges and for quantifying its effects, Antràs and Foley (2015) pose compelling evidence in favor of the existence of such a constraint.

At the face of a domestic positive productivity shock, response functions indicate that terms of trade from the perspective of the local economy appreciate temporarily. The shock is also followed by a sudden increase in the interest rate on intra period loans demanded by local firms to fund their imported inputs, which later on exhibits a prolonged transitory decline. When working capital requirements for imported inputs are higher, the adjustments in the working capital cost are sharper. Consequently, this leads to inputs recomposition in firms of the domestic economy which at the margin demand more imported inputs vis-à-vis domestic ones when working capital requirements are lower. Due to general equilibrium effects allocations in the  $F$ -economy are also affected.

Although I recognize that two country models such as those of the New Open Economy Macroeconomics and International Real Business Cycles literature have typically been used to study economies with similar characteristics, usually advanced ones, I have departed from the SOE framework to study non-symmetric economies like the Mexican and the U.S. economies given their production integration which in turn may require considering key prices such as the terms of trade as endogenous. This, at the light of the model presented here and as a first pass means trade in consumption and, crucially, inputs, where the latter is subject to a friction that is likely to be present in emerging market economies. This, in an environment of endogenous prices. Therefore, I have made an attempt to parameterize the model for the U.S. and Mexican economies with intrinsic limitations, and probably strong assumptions. In this sense, the results should be seen as merely illustrative. Further developments in this area are likely to provide better workhorse models to study so highly integrated economies while at the same time so different in many important economic aspects. Leading works in this dimension are those of Chen and Crucini (2016) and Rothert (2020).

In future work it is important to introduce nominal rigidities for quantitatively assessing monetary policy responses. In that line of work, incorporating a service sector could also be useful to generate a potential source of idiosyncratic monetary policy responses which in turn could have an impact on the working capital cost that I study in this paper. Finally, notice that the intermediate goods technology adopted in the model is very simple. Therefore, shifting from some inputs to others does not have productivity gains or losses. This is an area in which the model I pose cannot say anything.

## References

- AGUIAR, M., AND G. GOPINATH (2007): “Emerging market business cycles: The cycle is the trend,” *Journal of Political Economy*, 115(1), 69–102.
- AMITI, M., AND D. E. WEINSTEIN (2011): “Exports and financial shocks,” *The Quarterly Journal of Economics*, 126(4), 1841–1877.
- ANTRÀS, P., AND C. F. FOLEY (2015): “Poultry in motion: a study of international trade finance practices,” *Journal of Political Economy*, 123(4), 853–901.
- BACKUS, D. K., P. J. KEHOE, AND F. E. KYDLAND (1992): “International real business cycles,” *Journal of Political Economy*, 100(4), 745–775.
- (1995): “International business cycles: Theory vs. evidence,” in *Frontiers of Business Cycle Research*. Princeton University Press.
- BASU, S. (1995): “Intermediate goods and business cycles: Implications for productivity and welfare,” *American Economic Review*, 85(3), 512–31.
- BAXTER, M., AND M. J. CRUCINI (1995): “Business cycles and the asset structure of foreign trade,” *International Economic Review*, 36(4), 821–854.
- BERNANKE, B. S. (2015): “The Taylor Rule: A benchmark for monetary policy?,” *Ben Bernanke’s Blog*.
- BERNARD, A. B., J. EATON, J. B. JENSEN, AND S. KORTUM (2003): “Plants and productivity in international trade,” *American Economic Review*, 93(4), 1268–1290.
- BOUAKEZ, H., E. CARDIA, AND F. J. RUGE-MURCIA (2009): “The transmission of monetary policy in a multisector economy,” *International Economic Review*, 50(4), 1243–1266.
- CHARI, V. V., P. J. KEHOE, AND E. R. MCGRATTAN (2000): “Sticky price models of the business cycle: can the contract multiplier solve the persistence problem?,” *Econometrica*, 68(5), 1151–1179.
- (2007): “Business cycle accounting,” *Econometrica*, 75(3), 781–836.
- CHEN, K., AND M. J. CRUCINI (2016): “Trends and cycles in small open economies: making the case for a general equilibrium approach,” *Journal of Economic Dynamics and Control*, 72, 159–168.

- CHOR, D., AND K. MANOVA (2012): “Off the cliff and back? Credit conditions and international trade during the global financial crisis,” *Journal of International Economics*, 87(1), 117–133.
- CHOWDHURY, I., M. HOFFMANN, AND A. SCHABERT (2006): “Inflation dynamics and the cost channel of monetary transmission,” *European Economic Review*, 50(4), 995–1016.
- CHRISTIANO, L. J., AND M. EICHENBAUM (1995): “Liquidity effects, monetary policy, and the business cycle,” *Journal of Money, Credit and Banking*, 27(4), 1113–1136.
- CLARIDA, R., J. GALI, AND M. GERTLER (1999): “The science of monetary policy: a new Keynesian perspective,” *Journal of Economic Literature*, 37(4), 1661–1707.
- COLE, H. L., AND M. OBSTFELD (1991): “Commodity trade and international risk sharing: How much do financial markets matter?,” *Journal of Monetary Economics*, 28(1), 3–24.
- CORSETTI, G., L. DEDOLA, AND S. LEDUC (2008): “International risk sharing and the transmission of productivity shocks,” *The Review of Economic Studies*, 75(2), 443–473.
- DYRDA, S., G. KAPLAN, AND J.-V. RÍOS-RULL (2012): “Business cycles and household formation: The micro vs the macro labor elasticity,” NBER Working Paper 17880, Cambridge, MA.
- HEATHCOTE, J., AND F. PERRI (2002): “Financial autarky and international business cycles,” *Journal of Monetary Economics*, 49(3), 601–627.
- IDA, D. (2011): “Monetary policy and asset prices in an open economy,” *The North American Journal of Economics and Finance*, 22(2), 102–117.
- (2020): “Asset price stabilization and cost channel in a two-country new Keynesian model,” Discussion Paper Series No. 14, Department of Economics, Momoyama Gakuin University.
- ITSKHOKI, O., AND D. MUKHIN (2017): “Exchange rate disconnect in general equilibrium,” NBER Working Paper 23401, Cambridge, MA.
- (2021): “Exchange rate disconnect in general equilibrium,” *Journal of Political Economy*, 129(8), 2183–2232.
- JONES, C. I. (2011): “Intermediate goods and weak links in the theory of economic development,” *American Economic Journal: Macroeconomics*, 3(2), 1–28.

- KLAPPER, L., L. LAEVEN, AND R. RAJAN (2012): “Trade credit contracts,” *The Review of Financial Studies*, 25(3), 838–867.
- LANE, P. R. (2001): “The new open economy macroeconomics: a survey,” *Journal of International Economics*, 54(2), 235–266.
- LEYVA, G., AND C. URRUTIA (2020): “Informality, labor regulation, and the business cycle,” *Journal of International Economics*, 126, 103340.
- MENDOZA, E. G., AND V. Z. YUE (2012): “A general equilibrium model of sovereign default and business cycles,” *The Quarterly Journal of Economics*, 127(2), 889–946.
- MEZA, F., S. PRATAP, AND C. URRUTIA (2019): “Credit, misallocation and productivity growth: A disaggregated analysis,” *Review of Economic Dynamics*, 34, 61–86.
- NEUMEYER, P. A., AND F. PERRI (2005): “Business cycles in emerging economies: the role of interest rates,” *Journal of Monetary Economics*, 52(2), 345–380.
- PARAVISINI, D., V. RAPPOPORT, P. SCHNABL, AND D. WOLFENZON (2015): “Dissecting the effect of credit supply on trade: Evidence from matched credit-export data,” *The Review of Economic Studies*, 82(1), 333–359.
- PETERSEN, M. A., AND R. G. RAJAN (1995): “The effect of credit market competition on lending relationships,” *The Quarterly Journal of Economics*, 110(2), 407–443.
- RAMANARAYANAN, A. (2017): “Imported inputs, irreversibility, and international trade dynamics,” *Journal of International Economics*, 104, 1–18.
- ROTHERT, J. (2020): “International business cycles in emerging markets,” *International Economic Review*, 61(2), 753–781.
- SAMUELSON, P. A. (1954): “The transfer problem and transport costs, II: Analysis of effects of trade impediments,” *The Economic Journal*, 64(254), 264–289.
- SCHMIDT-EISENLOHR, T. (2013): “Towards a theory of trade finance,” *Journal of International Economics*, 91(1), 96–112.
- TAYLOR, J. B. (1993): “Discretion versus policy rules in practice,” in *Carnegie-Rochester conference series on public policy*, vol. 39, pp. 195–214. Elsevier.
- URIBE, M., AND V. Z. YUE (2006): “Country spreads and emerging countries: Who drives whom?,” *Journal of International Economics*, 69(1), 6–36.



# Appendix

## A.1 Derivation of $H$ -Economy Household First Order Conditions

Given  $B_{-1}, B_{-1}^*, Z_0^*, Z_0$  and  $\{e_t, W_t, i_t, i_t^*, i_t^d, P_t, \Pi_t, \{P_{H,t}(i)\}_{i \in [0,1]}, \{P_{F,t}^*(j)\}_{j \in [0,1]}\}$ , the representative household in the  $H$ -economy solves the following optimization problem:

$$\max_{\{c_{H,t}(i)\}_{i \in [0,1]}, \{c_{F,t}(j)\}_{j \in [0,1]}, l_t, D_t, B_t^*, B_t, Z_{t+1}^*, Z_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\varphi(\bar{l} - l_t)^{1+\chi}}{1+\chi} \right) \quad (\text{A.1})$$

s.t.

$$\begin{aligned} P_t c_t + \frac{D_t}{1+i_t^d} + e_t Z_{t+1}^* + Z_{t+1} + \frac{e_t B_t^*}{1+i_t^*} + \frac{B_t}{1+i_t} \\ = W_t l_t + D_t + e_t Z_t^* + Z_t + e_t B_{t-1}^* + B_{t-1} + \Pi_t \forall t \end{aligned} \quad (\text{A.2})$$

$$P_t c_t \leq e_t Z_t^* + Z_t - \frac{D_t}{1+i_t^d} \quad (\text{A.3})$$

where  $c_t$  is given by expression (4). Let  $-\beta^t \frac{\lambda_t}{P_t}$  and  $-\beta^t \frac{\mu_t}{P_t}$  be the Lagrange multipliers of constraints (A.2) and (A.3), respectively. The first order conditions of the problem are:

$$c_t^{-\sigma} = \lambda_t + \mu_t \quad (\text{A.4})$$

$$\varphi(\bar{l} - l_t)^\chi = \lambda_t \frac{W_t}{P_t} \quad (\text{A.5})$$

$$\mu_t = \lambda_t i_t^d \quad (\text{A.6})$$

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{e_{t+1}}{e_t} \frac{P_t}{P_{t+1}} \right] + \beta E_t \left[ \frac{\mu_{t+1}}{\lambda_t} \frac{e_{t+1}}{e_t} \frac{P_t}{P_{t+1}} \right] = 1 \quad (\text{A.7})$$

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right] + \beta E_t \left[ \frac{\mu_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right] = 1 \quad (\text{A.8})$$

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{e_{t+1}}{e_t} \frac{P_t}{P_{t+1}} (1+i_t^*) \right] = 1 \quad (\text{A.9})$$

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} (1 + i_t) \right] = 1 \quad (\text{A.10})$$

Using (A.4) and (A.5) we obtain

$$\frac{\varphi l_t^X}{c_t^{-\sigma}} = \frac{w_t}{1 + i_t^d} \quad (\text{A.11})$$

where  $w_t \equiv \frac{W_t}{P_t}$ . Using (A.7) multiplied by  $(1 + i_t^*)$  and equation (A.6) we obtain

$$i_t^d = i_{t-1}^* \forall t \quad (\text{A.12})$$

Similarly, using (A.8) multiplied by  $(1 + i_t)$  and equation (A.6) it is possible to obtain

$$i_t^d = i_{t-1} \forall t \quad (\text{A.13})$$

Therefore,

$$i_t^d = i_{t-1}^* = i_{t-1} \forall t \quad (\text{A.14})$$

Thus, using (A.4) and (A.12) we get

$$\lambda_t = \frac{c_t^{-\sigma}}{1 + i_{t-1}^*} \quad (\text{A.15})$$

Plugging (A.15) into (A.9) we obtain

$$\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{(1 + i_{t-1}^*) \left( \frac{e_{t+1}}{e_t} \right)}{\pi_{t+1}} \right] = 1 \quad (\text{A.16})$$

Moreover, plugging  $\lambda_t = \frac{c_t^{-\sigma}}{1 + i_{t-1}^*}$  into (A.10) we get

$$\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{(1 + i_{t-1})}{\pi_{t+1}} \right] = 1 \quad (\text{A.17})$$

Condition

$$P_t c_t = e_t Z_t^* + Z_t - \frac{D_t}{(1 + i_t^d)} \quad (\text{A.18})$$

must also hold every period. Equations (10) and (11) must also be satisfied from the intratemporal consumer's problem.

## A.2 Derivation of $H$ -Economy Firms' First Order Conditions

First, we obtain the first order conditions of the problem (27). For any  $t', t \in \mathbb{N} \cup 0$ ,  $t' \leq t$ , formulate the problem as taking input prices  $\left\{ w_t, \left\{ \frac{P_{H,t}^m(i)}{P_t} \right\}_{i \in [0,1]}, \left\{ \frac{e_t P_{F,t}^{m*}(j)}{P_t} \right\}_{j \in [0,1]} \right\}$  and  $\{Q_t, e_t, i_t^L\}$  as given to solve:

$$\left\{ \begin{array}{l} \max \\ L_t(i), \frac{P_{H,t}(i)}{P_t}, \\ \frac{P_{H,t}^m(i)}{P_t}, l_t(i), \\ \{m_{H,t}(i)\}_{i \in [0,1]}, \\ \{m_{F,t}(j)\}_{j \in [0,1]} \end{array} \right\} E_{t'} \sum_{t=t'}^{\infty} d_{t',t} \left[ \begin{array}{l} \frac{L_t(i)}{(1+i_t^L)P_t} - \frac{L_t(i)}{P_t} \\ + \mu_H \left( \frac{P_{H,t}(i)}{P_t} Y_{c,H,t}(i) + \frac{P_{H,t}^m(i)}{P_t} Y_{m,H,t}(i) \right) \\ + \mu_F Q_t \left( \frac{P_{H,t}(i)}{e_t P_t^*} Y_{c,H,t}^*(i) + \frac{P_{H,t}^m(i)}{e_t P_t^*} Y_{m,H,t}^*(i) \right) \\ - w_t l_t(i) - \int_0^1 \frac{P_{H,t}^m(i)}{P_t} m_{H,t}(i) di \\ - \int_0^1 \frac{e_t P_{F,t}^{m*}(j)}{P_t} m_{F,t}(j) dj \end{array} \right] \quad (\text{A.19})$$

s.t.

$$\int_0^1 v e_t P_{F,t}^{m*}(j) m_{F,t}(j) dj \leq \frac{L_t(i)}{1+i_t^L} \quad (\text{A.20})$$

$$\mu_H(Y_{c,H,t}(i) + Y_{m,H,t}(i)) + \mu_F(Y_{c,H,t}^*(i) + Y_{m,H,t}^*(i)) \leq e^{z_t} M_t(i)^{\phi_H} l_t(i)^{1-\phi_H} \quad (\text{A.21})$$

with  $M_t(i) = \left( \int_0^1 \left[ (1-\omega)^{\frac{1}{\eta}} m_{H,t}(i)^{\frac{\eta-1}{\eta}} + \omega^{\frac{1}{\eta}} m_{F,t}(i)^{\frac{\eta-1}{\eta}} \right] di \right)^{\frac{\eta}{\eta-1}}$ ,  $z_t = (1-\rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z$ ,  $\varepsilon_t^z \sim iid N(0, \sigma_{\varepsilon^z})$  with  $|\rho_z| < 1$  and where demand functions are given by  $Y_{c,H,t}(i) \equiv c_{H,t}(i) = (1-\omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\eta} c_t$ ,  $Y_{m,H,t}(i) \equiv m_{H,t}(i) = (1-\omega) \left( \frac{P_{H,t}^m(i)}{P_{M,t}} \right)^{-\eta} M_t$ ,  $Y_{c,H,t}^*(i) \equiv c_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}(i)}{e_t P_t^*} \right)^{-\eta} c_t^*$  and  $Y_{m,H,t}^*(i) \equiv m_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}^m(i)}{e_t P_t^*} \right)^{-\eta} M_t^*$ .

Notice that since intra period loans are costly, constraint (A.20) holds with equality, and

we can reexpress the above problem as

$$\left\{ \begin{array}{l} \max \\ \frac{P_{H,t}(i)}{P_t}, \\ \frac{P_{H,t}^m(i)}{P_t}, l_t(i), \\ \{m_{H,t}(i)\}_{i \in [0,1]}, \\ \{m_{F,t}(j)\}_{j \in [0,1]} \end{array} \right\} E_{t'} \sum_{t=t'}^{\infty} d_{t',t} \left[ \begin{array}{l} \mu_H \left( \frac{P_{H,t}(i)}{P_t} Y_{c,H,t}(i) + \frac{P_{H,t}^m(i)}{P_t} Y_{m,H,t}(i) \right) \\ + \mu_F Q_t \left( \frac{P_{H,t}(i)}{e_t P_t^*} Y_{c,H,t}^*(i) + \frac{P_{H,t}^m(i)}{e_t P_t^*} Y_{m,H,t}^*(i) \right) \\ - w_t l_t(i) - \int_0^1 \frac{P_{H,t}^m(i)}{P_t} m_{H,t}(i) di \\ - (1 + \nu i_t^L) \int_0^1 \frac{e_t P_{F,t}^m(j)}{P_t} m_{F,t}(j) dj \end{array} \right] \quad (\text{A.22})$$

s.t.

$$\mu_H (Y_{c,H,t}(i) + Y_{m,H,t}(i)) + \mu_F (Y_{c,H,t}^*(i) + Y_{m,H,t}^*(i)) \leq e^{z_t} M_t(i)^{\phi_H} l_t(i)^{1-\phi_H} \quad (\text{A.23})$$

with  $M_t(i) = \left( \int_0^1 \left[ (1-\omega)^{\frac{1}{\eta}} m_{H,t}(i)^{\frac{\eta-1}{\eta}} + \omega^{\frac{1}{\eta}} m_{F,t}(i)^{\frac{\eta-1}{\eta}} \right] di \right)^{\frac{\eta}{\eta-1}}$ ,  $z_t = (1-\rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z$ ,  $\varepsilon_t^z \sim iid N(0, \sigma_{\varepsilon^z})$  with  $|\rho_z| < 1$  and where demand schedules are given by  $Y_{c,H,t}(i) \equiv c_{H,t}(i) = (1-\omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\eta} c_t$ ,  $Y_{m,H,t}(i) \equiv m_{H,t}(i) = (1-\omega) \left( \frac{P_{H,t}^m(i)}{P_{M,t}} \right)^{-\eta} M_t$ ,  $Y_{c,H,t}^*(i) \equiv c_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}(i)}{e_t P_t^*} \right)^{-\eta} c_t^*$  and  $Y_{m,H,t}^*(i) \equiv m_{H,t}^*(i) = \omega^* \left( \frac{P_{H,t}^m(i)}{e_t P_t^*} \right)^{-\eta} M_t^*$ .

Let  $-d_{t',t} \gamma_t(i)$  be the Lagrange multiplier for constraint (A.23). The first order conditions of the problem are:

$$\left( \frac{\eta-1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} = \gamma_t(i) \quad (\text{A.24})$$

$$\left( \frac{\eta-1}{\eta} \right) \frac{P_{H,t}^m(i)}{P_t} = \gamma_t(i) \quad (\text{A.25})$$

$$1 = \left( \frac{\eta-1}{\eta} \right) \phi_H e^{z_t} M_t(i)^{\phi_H - 1 + \frac{1}{\eta}} l_t(i)^{1-\phi_H} \times (1-\omega)^{\frac{1}{\eta}} m_{H,t}^{-\frac{1}{\eta}}(i) \quad (\text{A.26})$$

$$(1 + \nu i_t^L) \frac{e_t P_{F,t}^m(i)}{P_t} = \gamma_t(i) \phi_H e^{z_t} M_t(i)^{\phi_H - 1 + \frac{1}{\eta}} l_t(i)^{1-\phi_H} \times \omega^{\frac{1}{\eta}} m_{F,t}^{-\frac{1}{\eta}}(i) \quad (\text{A.27})$$

$$w_t = \gamma_t(i) (1 - \phi_H) e^{z_t} M_t(i)^{\phi_H} l_t(i)^{-\phi_H} \quad (\text{A.28})$$

Using (A.24) and (A.25) we have

$$\frac{P_{H,t}(i)}{P_t} = \frac{P_{H,t}^m(i)}{P_t} = \frac{P_{H,t}}{P_t} \quad \forall i \in [0, 1] \quad (\text{A.29})$$

where the latter equality follows from the fact that all firms  $i \in [0, 1]$  in the  $H$ -economy are identical.

Plugging (A.24) into (A.28), it is straightforward to obtain

$$w_t = \left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} (1 - \phi_H) e^{z_t} M_t(i)^{\phi_H} l_t(i)^{-\phi_H} \quad (\text{A.30})$$

Define  $T_t \equiv \frac{e_t P_{F,t}^*}{P_{H,t}}$  and  $wk_t \equiv \nu i_t^L$ . Now, use the fact that  $P_{H,t} = P_{H,t}^m$  (and the analogous relation for price  $F$ ) and plugging (A.24) into (A.27) we obtain

$$T_t(1 + wk_t) = \left( \frac{\eta - 1}{\eta} \right) \phi_H e^{z_t} M_t(i)^{\phi_H - 1 + \frac{1}{\eta}} l_t(i)^{1 - \phi_H} \times \omega^{\frac{1}{\eta}} m_{F,t}^{-\frac{1}{\eta}}(i) \quad (\text{A.31})$$

Further, we can divide (A.31) by (A.26) to get

$$T_t(1 + wk_t) = \frac{\omega^{\frac{1}{\eta}} m_{F,t}(i)^{-\frac{1}{\eta}}}{(1 - \omega)^{\frac{1}{\eta}} m_{H,t}(i)^{-\frac{1}{\eta}}} \quad (\text{A.32})$$

Given that the firm  $i$  faces the same prices for their inputs,  $m_{H,t}(i) = m_{H,t}$  and  $m_{F,t}(j) = m_{F,t}$ , and therefore, the above condition can be expressed as

$$\frac{m_{F,t}}{m_{H,t}} = \frac{\omega}{(1 - \omega)} (T_t(1 + wk_t))^{-\eta} \quad (\text{A.33})$$

Moreover, using (A.24) and expressions (A.26) and (A.27), after some algebraic manipulations one obtains

$$\begin{aligned} & \phi_H \left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} \\ &= \frac{M_t(i)}{Y_t(i)} \left( \int_0^1 \left[ (1 - \omega) \left( \frac{P_{H,t}(i)}{P_t} \right)^{1 - \eta} + \omega \left( (1 + wk_t) \frac{e_t P_{F,t}^*(i)}{P_t} \right)^{1 - \eta} \right] di \right)^{\frac{1}{1 - \eta}} \end{aligned} \quad (\text{A.34})$$

and therefore,

$$\phi_H \left( \frac{\eta - 1}{\eta} \right) \frac{P_{H,t}(i)}{P_t} = \frac{M_t(i)}{Y_t(i)} \frac{P_{M,t}}{P_t} \quad (\text{A.35})$$

Finally, observe that from equations (A.30) and (A.35), it follows that

$$\left(\frac{\eta-1}{\eta}\right) \frac{P_{H,t}(i)}{P_t} Y_t(i) = \frac{P_{M,t}}{P_t} M_t(i) + w_t l_t(i) = mc_t Y_t(i) \quad (\text{A.36})$$

where  $mc_t = e^{-z_t} \left(\frac{w_t}{1-\phi_H}\right)^{1-\phi_H} \left(\frac{P_{M,t}}{P_t \phi_H}\right)^{\phi_H}$ . Notice that the previous condition implies that  $\left(\frac{\eta-1}{\eta}\right) \frac{P_{H,t}}{P_t} = mc_t$ .

### A.3 Derivation of $F$ -Economy Firms' First Order Conditions

For any  $t', t \in \mathbb{N} \cup 0, t' \leq t$ , formulate the problem as taking input prices

$$\left\{ w_t^*, \left\{ \frac{P_{H,t}^m(i)}{e_t P_t^*} \right\}_{i \in [0,1]}, \left\{ \frac{P_{F,t}^{m^*}(j)}{P_t^*} \right\}_{j \in [0,1]} \right\}, \{Q_t, e_t, i_t^{*d}, D_t^*(j)\} \text{ and demand functions}$$

$$Y_{c,F,t}(j) \equiv c_{F,t}(j) = \omega \left( \frac{e_t P_{F,t}^*(j)}{P_t} \right)^{-\eta} c_t, Y_{m,F,t}(j) \equiv m_{F,t}(j) = \omega \left( \frac{(1+wk_t)e_t P_{F,t}^{m^*}(j)}{P_{M,t}} \right)^{-\eta} M_t,$$

$$Y_{c,F,t}^*(j) \equiv c_{F,t}^*(j) = (1-\omega^*) \left( \frac{P_{F,t}^*(j)}{P_t^*} \right)^{-\eta} c_t^* \text{ and } Y_{m,F,t}^*(j) \equiv m_{F,t}^*(j) = (1-\omega^*) \left( \frac{P_{F,t}^{m^*}(j)}{P_t^*} \right)^{-\eta} M_t^*$$

as given, the firm  $j \in [0, 1]$  solves:

$$\left\{ \begin{array}{l} \max \\ \frac{P_{F,t}^*(j)}{P_t^*}, \frac{P_{F,t}^{m^*}(j)}{P_t^*}, \\ l_t(j), \{m_{H,t}(i)\}_{i \in [0,1]}, \\ \{m_{F,t}(j)\}_{j \in [0,1]} \end{array} \right\} E_{t'} \sum_{t=t'}^{\infty} d_{t',t}^* \left[ \begin{array}{l} \mu_H \frac{1}{Q_t} \left( \frac{e_t P_{F,t}^*(j)}{P_t} Y_{c,F,t}(j) + \frac{e_t P_{F,t}^{m^*}(j)}{P_t} Y_{m,F,t}(j) \right) \\ + \mu_F \left( \frac{P_{F,t}^*(j)}{P_t^*} Y_{c,F,t}^*(j) + \frac{P_{F,t}^{m^*}(j)}{P_t^*} Y_{m,F,t}^*(j) \right) \\ - w_t^* l_t^*(j) - \int_0^1 \frac{P_{H,t}^m(i)}{e_t P_t^*} m_{H,t}^*(i) di \\ - \int_0^1 \frac{P_{F,t}^{m^*}(j)}{P_t^*} m_{F,t}^*(j) dj - \frac{D_t^*(j)}{(1+i_t^{*d})P_t^*} + \frac{D_t^*(j)}{P_t^*} \end{array} \right] \quad (\text{A.37})$$

s.t.

$$\mu_H(Y_{c,F,t}(j) + Y_{m,F,t}(j)) + \mu_F(Y_{c,F,t}^*(j) + Y_{m,F,t}^*(j)) \leq e^{z_t^*} M_t(j)^{\phi_F} l_t(j)^{1-\phi_F} \quad (\text{A.38})$$

with  $M_t^*(j) = \left( \int_0^1 \left[ \omega^{*\frac{1}{\eta}} m_{H,t}^{*\frac{\eta-1}{\eta}}(j) + (1-\omega^*)^{\frac{1}{\eta}} m_{F,t}^{*\frac{\eta-1}{\eta}}(j) \right] dj \right)^{\frac{\eta}{\eta-1}}$  and  $z_t^* = (1-\rho_{z^*})z^* + \rho_{z^*} z_{t-1}^* + \varepsilon_t^{z^*}$ ,  $\varepsilon_t^{z^*} \sim iid N(0, \sigma_{\varepsilon_{z^*}})$  with  $|\rho_{z^*}| < 1$  and  $\frac{D_t^*(j)}{(1+i_t^{*d})P_t^*} = \frac{\mu_H}{\mu_F} \int_0^1 \frac{Z_t^{*f}(i)}{P_t^*} di$ , for any  $j \in [0, 1]$  which is the exporting firm in the  $F$ -economy and for  $i \in [0, 1]$  which are the importing ones in the  $H$ -economy.

Since we assume that all firms are symmetric and, therefore, all of them import, the integral is over the whole set of importing firms. Notice that the principal of the deposit equals the amount of the payment in advance in foreign currency, scaled down to account for the different sizes of the economies. Given that this is an intra period deposit, at the end of the period the firm  $j \in [0, 1]$  will receive from the financial intermediary  $D_t^*(j) = \frac{\mu_H}{\mu_F} \int_0^1 Z_t^{*f}(i)(1+i_t^{*d})di$ . From first order conditions of the foreign household we have that  $i_t^{*d} = i_{t-1}^*$  and from those

of the domestic household we know that  $i_t^L = i_{t-1}^*$ , hence  $D_t^*(j) = \frac{\mu_H}{\mu_F} \int_0^1 Z_t^{*f}(i)(1 + i_t^L)di$ . This is precisely the amount of foreign currency the exporting firm  $j$  requires giving back to a particular importing firm  $i \in [0, 1]$ , that we assume is randomly assigned to, which scaled up for economies size totals  $\int_0^1 Z_t^{*f}(i)(1 + i_t^L)di = (1 + i_t^L)Z_t^{*f}(i)$ .

Let  $-d_{v,t}^* \gamma_t^*(j)$  be the Lagrange multiplier for constraint (A.38). The first order conditions of the problem are:

$$\left(\frac{\eta - 1}{\eta}\right) \frac{P_{F,t}^*(j)}{P_t^*} = \gamma_t^*(j) \quad (\text{A.39})$$

$$\left(\frac{\eta - 1}{\eta}\right) \frac{P_{F,t}^{m*}(j)}{P_t^*} = \gamma_t^*(j) \quad (\text{A.40})$$

$$1 = \left(\frac{\eta - 1}{\eta}\right) \phi_F e^{z_t^*} M_t(j)^{* \phi_F - 1 + \frac{1}{\eta}} l_t(j)^{* 1 - \phi_F} \times (1 - \omega^*)^{\frac{1}{\eta}} m_{F,t}(j)^{* - \frac{1}{\eta}} \quad (\text{A.41})$$

$$\frac{P_{H,t}^m(j)}{e_t P_t^*} = \gamma_t^*(j) \phi_F e^{z_t^*} M_t(j)^{* \phi_F - 1 + \frac{1}{\eta}} l_t(j)^{* 1 - \phi_F} \times \omega^{*\frac{1}{\eta}} m_{H,t}(j)^{* - \frac{1}{\eta}} \quad (\text{A.42})$$

$$w_t^* = \gamma_t^*(j) (1 - \phi_F) e^{z_t^*} M_t(j)^{* \phi_F} l_t(j)^{* - \phi_F} \quad (\text{A.43})$$

Using (A.39) and (A.43) it is straightforward to obtain

$$w_t^* = \left(\frac{\eta - 1}{\eta}\right) \frac{P_{F,t}^*(j)}{P_t^*} (1 - \phi_F) e^{z_t^*} M_t(j)^{* \phi_F} l_t(j)^{* - \phi_F} \quad (\text{A.44})$$

Dividing (A.41) by (A.42), using equation (A.39) and (A.40) and given that firm  $j$  faces the same prices for its inputs  $m_{F,t}^*(j) = m_{F,t}^*$  and  $m_{H,t}^*(j) = m_{H,t}^*$ , then we get

$$T_t = \frac{(1 - \omega^*)^{\frac{1}{\eta}} m_{F,t}^{*\frac{-1}{\eta}}}{\omega^{*\frac{1}{\eta}} m_{H,t}^{*\frac{-1}{\eta}}} \quad (\text{A.45})$$

Moreover, since  $P_{F,t} = P_{F,t}^m$  (and  $P_{H,t} = P_{H,t}^m$ ), as in the case of the  $H$ -economy firm  $i$



problem, we can reach the following condition:

$$\phi_F \left( \frac{\eta - 1}{\eta} \right) \frac{P_{F,t}^*(j)}{P_t^*} = \frac{M_t^*(j)}{Y_t^*(j)} \left( \int_0^1 \left[ (1 - \omega^*) \left( \frac{P_{F,t}^*(j)}{P_t^*} \right)^{1-\eta} + \omega^* \left( \frac{P_{H,t}(j)}{e_t P_t^*} \right)^{1-\eta} \right] dj \right)^{\frac{1}{1-\eta}} \quad (\text{A.46})$$

and therefore,

$$\phi_F \left( \frac{\eta - 1}{\eta} \right) \frac{P_{F,t}^*(j)}{P_t^*} = \frac{M_t^*(j)}{Y_t^*(j)} \quad (\text{A.47})$$

Finally, observe that from equations (A.43) and (A.47), it follows that

$$\left( \frac{\eta - 1}{\eta} \right) \frac{P_{F,t}^*(j)}{P_t^*} Y_t^*(j) = M_t^*(j) + w_t^* l_t^*(j) = mc_t^* Y_t^*(j) \quad (\text{A.48})$$

where  $mc_t^* = e^{-z_t^*} \left( \frac{w_t^*}{1-\phi_F} \right)^{1-\phi_F} \left( \frac{1}{\phi_F} \right)^{\phi_F}$ . Notice that the previous condition implies that

$$\left( \frac{\eta - 1}{\eta} \right) \frac{P_{F,t}^*}{P_t^*} = mc_t^*$$

## A.4 Derivation of the Two-Country Economy Feasibility Constraint

Take the budget constraint of the representative household of the  $H$ -economy expressed in real terms

$$\begin{aligned} c_t + \frac{D_t}{(1+i_t^d)P_t} + \frac{e_t Z_{t+1}^*}{P_t} + \frac{Z_{t+1}}{P_t} + \frac{e_t B_t^*}{(1+i_t^*)P_t} + \frac{B_t}{(1+i_t)P_t} \\ = w_t l_t + \frac{D_t}{P_t} + \frac{e_t Z_t^*}{P_t} + \frac{Z_t}{P_t} + \frac{e_t B_{t-1}^*}{P_t} + \frac{B_{t-1}}{P_t} + \frac{\Pi_t}{P_t} \forall t \end{aligned} \quad (\text{A.49})$$

and consider the consolidated  $H$ -economy government budget constraint

$$\frac{Z_{t+1} - Z_t}{P_t} + \frac{B_t}{(1+i_t)P_t} - \frac{B_{t-1}}{P_t} = 0 \quad (\text{A.50})$$

where we are using the fact that  $Z_t = M_t^s$ , where  $M_t^s$  is the money supply. Hence, using (A.50) into (A.49) we obtain

$$c_t + \frac{D_t}{(1+i_t^d)P_t} + \frac{e_t Z_{t+1}^*}{P_t} + \frac{e_t B_t^*}{(1+i_t^*)P_t} = w_t l_t + \frac{D_t}{P_t} + \frac{e_t Z_t^*}{P_t} + \frac{e_t B_{t-1}^*}{P_t} + \frac{\Pi_t}{P_t} \quad (\text{A.51})$$

Now consider the budget constraint of the firm  $i \in [0, 1]$  of the  $H$ -economy expressed in real terms

$$\frac{\Pi_t(i)}{P_t} = \frac{P_{H,t}(i)}{P_t} Y_t(i) - M_t(i) - w_t l_t(i) - \frac{i_t^L}{(1+i_t^L)} \frac{L_t(i)}{P_t} \quad (\text{A.52})$$

Since  $\frac{\Pi_t}{P_t} \equiv \int_0^1 \frac{\Pi_t(i)}{P_t} di$ , given that prices and allocations are identical across firms  $i \in [0, 1]$ , we have that

$$\frac{\Pi_t}{P_t} \equiv \int_0^1 \frac{\Pi_t(i)}{P_t} di = \frac{P_{H,t}}{P_t} Y_t - M_t - w_t l_t - \frac{i_t^L}{(1+i_t^L)} \frac{L_t}{P_t} \quad (\text{A.53})$$

Plugging (A.53) into (A.51) and using the equilibrium condition that  $D_t = L_t$  we obtain

$$c_t + M_t + \frac{e_t Z_{t+1}^*}{P_t} - \frac{e_t Z_t^*}{P_t} + \frac{e_t B_t^*}{(1+i_t^*)P_t} = \frac{e_t B_{t-1}^*}{P_t} + \frac{P_{H,t}}{P_t} Y_t \forall t \quad (\text{A.54})$$

Observe that we can express the previous equation as

$$\frac{B_t^*}{(1+i_t^*)P_t^*} - \frac{B_{t-1}^*}{P_t^*} + \frac{(Z_{t+1}^* - Z_t^*)}{P_t^*} = \frac{1}{Q_t} \left( \frac{P_{H,t}}{P_t} Y_t - c_t - M_t \right) \quad (\text{A.55})$$

Now consider the budget of the  $F$ -economy household

$$c_t^* + \frac{\tilde{B}_t^*}{(1+i_t^*)P_t^*} + \frac{\tilde{Z}_{t+1}^*}{P_t^*} - \frac{L_t^*}{(1+i_t^{*L})P_t^*} = w_t^*l_t^* + \frac{\tilde{B}_{t-1}^*}{P_t^*} + \frac{\tilde{Z}_t^*}{P_t^*} - \frac{L_t^*}{P_t^*} + \frac{\Pi_t^*}{P_t^*} \quad (\text{A.56})$$

and also consider aggregate profits of  $F$ -economy firms, which are identical, expressed in real terms

$$\frac{\Pi_t^*}{P_t^*} = \frac{P_{F,t}^*}{P_t^*}Y_t^* - \frac{P_{H,t}}{e_tP_t^*}m_{H,t}^* - \frac{P_{F,t}^*}{P_t^*}m_{F,t}^* - w_t^*l_t^* + \frac{i_t^{*d}}{(1+i_t^{*d})} \frac{D_t^*}{P_t^*} \quad (\text{A.57})$$

Since  $M_t^* = \frac{P_{H,t}}{e_tP_t^*}m_{H,t}^* + \frac{P_{F,t}^*}{P_t^*}m_{F,t}^*$ , plugging (A.57) into the budget constraint of the  $F$ -economy household, equation (A.56) and using the equilibrium condition that  $L_t^* = D_t^*$ , it can be re-expressed as

$$c_t^* + M_t^* + \frac{\tilde{B}_t^*}{(1+i_t^*)P_t^*} + \frac{\tilde{Z}_{t+1}^*}{P_t^*} = \frac{\tilde{B}_{t-1}^*}{P_t^*} + \frac{\tilde{Z}_t^*}{P_t^*} + \frac{P_{F,t}^*}{P_t^*}Y_t^* \quad (\text{A.58})$$

Rearranging terms we obtain

$$\frac{\tilde{B}_t^*}{(1+i_t^*)P_t^*} - \frac{\tilde{B}_{t-1}^*}{P_t^*} + \frac{(\tilde{Z}_{t+1}^* - \tilde{Z}_t^*)}{P_t^*} = \frac{P_{F,t}^*}{P_t^*}Y_t^* - c_t^* - M_t^* \quad (\text{A.59})$$

We can multiply (A.55) by  $\mu_H$  and (A.59) by  $\mu_F$  to obtain

$$\begin{aligned} & \mu_H \left( \frac{B_t^*}{(1+i_t^*)P_t^*} - \frac{B_{t-1}^*}{P_t^*} \right) + \mu_H \left( \frac{Z_{t+1}^* - Z_t^*}{P_t^*} \right) + \mu_F \left( \frac{\tilde{B}_t^*}{(1+i_t^*)P_t^*} - \frac{\tilde{B}_{t-1}^*}{P_t^*} \right) + \mu_F \left( \frac{\tilde{Z}_{t+1}^* - \tilde{Z}_t^*}{P_t^*} \right) \\ & = \mu_H \frac{1}{Q_t} \left( \frac{P_{H,t}}{P_t} Y_t - c_t - M_t \right) + \mu_F \left( \frac{P_{F,t}^*}{P_t^*} Y_t^* - c_t^* - M_t^* \right) \end{aligned} \quad (\text{A.60})$$

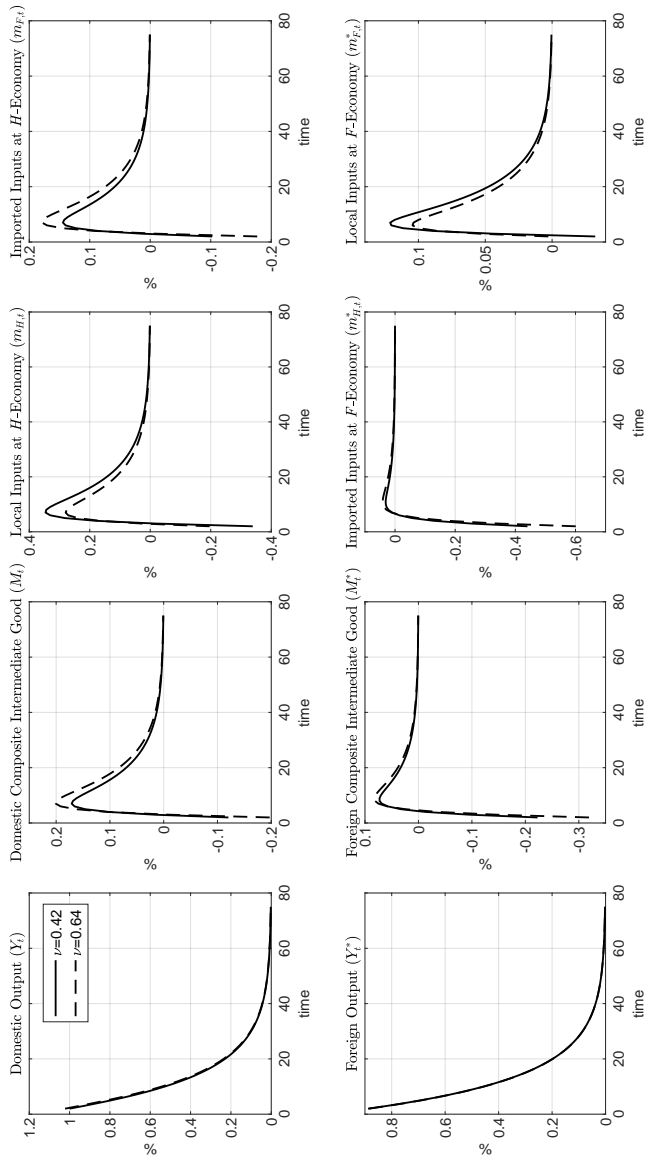
The budget constraint of the government of the  $F$  economy must satisfy

$$\frac{M_{t+1}^{*s} - M_t^{*s}}{P_t^*} + \mu_H \left( \frac{B_t^*}{(1+i_t^*)P_t^*} - \frac{B_{t-1}^*}{P_t^*} \right) + \mu_F \left( \frac{\tilde{B}_t^*}{(1+i_t^*)P_t^*} - \frac{\tilde{B}_{t-1}^*}{P_t^*} \right) = 0 \quad (\text{A.61})$$

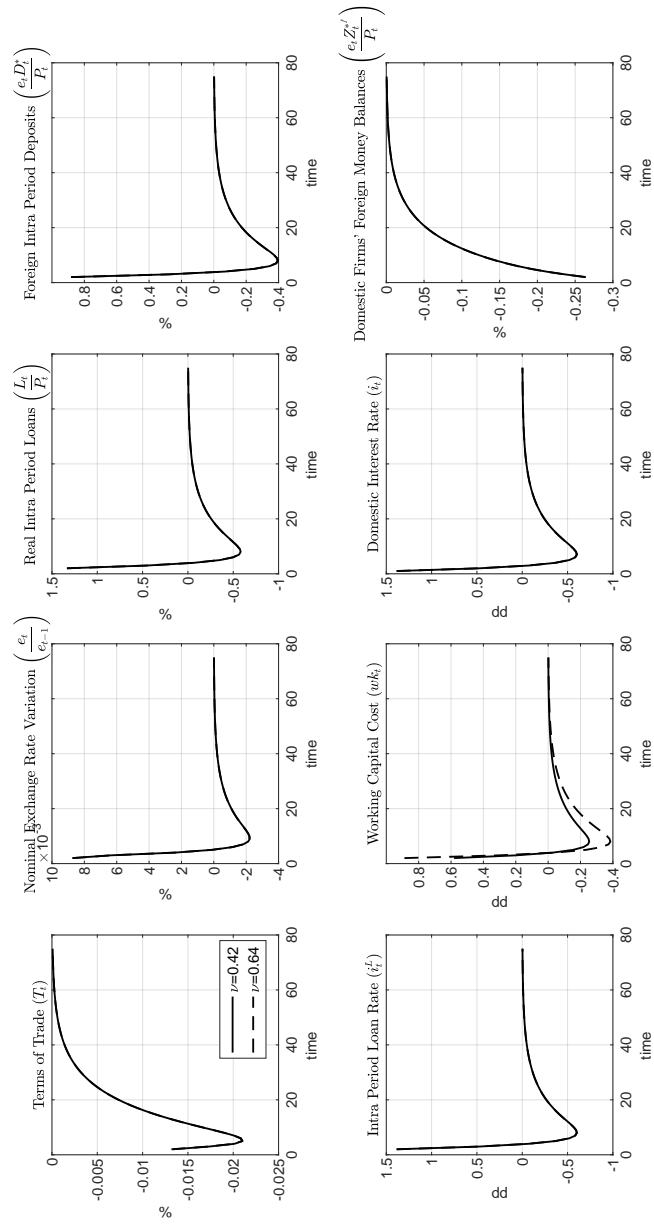
where  $M_t^{*s}$  is the foreign money supply, which in this setting equals to  $M_t^{*s} = \mu_H Z_t^* + \mu_F \tilde{Z}_t^*$ . Therefore, it follows that

$$\mu_H (c_t + M_t) + \mu_F Q_t (c_t^* + M_t^*) = \mu_H \frac{P_{H,t}}{P_t} Y_t + \mu_F Q_t \frac{P_{F,t}^*}{P_t^*} Y_t^* \quad (\text{A.62})$$

which represents the two-country economy feasibility constraint.



**Figure 1: Response to a positive productivity shock of the domestic economy that increases output by 1%**



**Figure 1 (cont.): Response to a positive productivity shock of the domestic economy that increases output by 1%**